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Specialization, Productivity and Financing Constraints

Abstract

We analyze optimal financial contracts when the specificity of investments is endogenous. Specialization decreases the liquidation value of assets, but it also improves the project’s long term productivity. While the first effect is known to make financing more difficult, we show that the second effect can ease financing constraints and increase financing capacity by improving an entrepreneur’s incentive to repay. The overall impact of specialization on the terms of financing depends on which effect is more important. We provide several new testable predictions about how a firm’s specialization decision interacts with the nature of investments, their timing, need for outside financing, and an entrepreneur’s ability to commit to a level of specialization.

Keywords: specialization, liquidation, incomplete contracts, asset specificity, debt, optimal contracts, staging investments

JEL Classification: G32, D23, G24.
1 Introduction

Investments in specialized assets constitute an important part of economic activity (see, e.g., Williamson, 1981). Moreover, such assets are generally viewed as highly productive if used within the firm. However, specialized assets generally have lower liquidation values since their productivity for alternative uses is relatively poor. As a result, financing specialized assets can be difficult to the extent that the ability to obtain outside financing may depend critically on an asset’s liquidation value. This is particularly true for entrepreneurial firms, where imperfect verifiability of cash flows may force financiers to rely on control rights and liquidation threats to ensure repayment (see, e.g., Aghion and Bolton, 1992, Hart and Moore, 1994, Bolton and Scharfstein, 1990, 1996).

Prior literature focusing on the importance of liquidation value has taken the degree of asset specificity as exogenous. However, the decision of what assets to acquire and how specialized they should be is often an integral component of a firm’s investment decision. This decision is particularly important when specialization is not easily contractible as the entrepreneur may then have discretion in determining asset specificity after obtaining financing. Such specialization is prevalent in high technology industries, where innovation and R&D activities naturally make use of specialized physical assets and human capital. Specific investments are also seen in manufacturing, where parts suppliers may need to produce tailored products for their downstream customers and often require the use of specialized equipment (see, e.g., Klein, Crawford, and Alchian, 1978).

Difficulties in specifying in advance the exact nature of the assets required, high costs of overseeing investment policy, and a limited ability or incentive to monitor all suggest that investors should consider an entrepreneur’s incentives to specialize assets when determining the terms of financial contracts. Likewise, entrepreneurs should take into account the

1See Diamond (1984), Ramakrishnan and Thakor (1984), Besanko and Kanatas (1993), Chemmanur and
availability and terms of outside financing in their specialization decisions. There is thus a clear interaction between asset specialization, financing, and investment which has been little explored.

We present a theory of endogenous asset specificity when firms are financially constrained. In our model, an entrepreneur requires financing to invest in a two-period project whose cash flows are not verifiable by a third party and therefore cannot be contracted upon. Hence, obtaining financing requires providing the investor with liquidation rights over the project’s assets. The innovation we introduce is that the entrepreneur can choose how much to specialize the project’s assets. This action is not contractible and is costly. Specialization increases the long term productivity of assets within the firm, but it also reduces their value under liquidation if the project is terminated. We find that, as in other financial contracting models (see, e.g., Faure-Grimaud, 2000), a contract resembling debt is optimal for financing entrepreneurs’ investments. This contract specifies a payment to the investor and a probability of continuation as a function of the reported interim cash flows.

Our contribution is to analyze how specialization affects this optimal contract. The entrepreneur’s ability to choose the degree of specialization affects the ex ante terms of the optimal contract through two distinct channels. First, anticipating that specialization may erode the liquidation value of assets, investors demand higher repayment as well as greater liquidation rights. Second, however, the increase in long term cash flows resulting from specialization increases the entrepreneur’s incentive to continue the project and makes him more willing to repay investors in order to avoid liquidation. This is reflected in the financial contract through an increase in the promised repayment, but a decrease in the probability of liquidation. To the best of our knowledge, this effect has not been studied, and it implies that investors may require less of a threat to convince the entrepreneur to pay since the Fulghieri (1994), and Rajan and Winton (1995) for models with a role for monitoring by a financier.
higher long term cash flows provide the entrepreneur with a superior bonding mechanism.

The overall effect of specialization on the ex ante contract terms and financing capacity of a project thus depends on how specialization affects liquidation value relative to long term cash flows. A novel implication of our analysis is that specialization may in fact increase the financing capacity of a project through its effect on long term cash flows, in contrast to the more commonly asserted notion that specialization, by reducing liquidation value, necessarily makes financing more difficult to obtain (Williamson, 1988). The need to obtain outside financing, however, leads to an inefficiently low level of specialization compared to the first best level achieved under self financing. This occurs because investors’ liquidation threat prevents the entrepreneur from fully benefiting from the improved productivity arising from specialization. Therefore, projects requiring more external financing, i.e., that are more levered, will have less specialized assets.

We also show that an entrepreneur’s inability to credibly commit to a particular level of specialization makes him worse off and may result in over or under specialization relative to what he would choose if he could commit. The inability to commit results in greater specialization when specialization has a relatively large effect on liquidation value and less specialization when specialization has a relatively large effect on long term cash flows. This is because the entrepreneur does not internalize how specialization affects financing terms when unable to commit.

We study two important extensions. First, we analyze the case when the cost of specializing assets is pecuniary and has to be financed up-front by the investor, and show that the entrepreneur chooses a lower level of specialization compared to the case when specialization costs are non-pecuniary. Second, we study how the specialization decision is affected when the entrepreneur can invest in two stages as in Bolton and Scharfstein (1990). Delaying investment lowers interim cash flows but also lowers the expected cost of investment because
when the project is liquidated the portion of investment that was delayed is not made. We show that when delaying some of the investment is optimal, the continuation probability increases, which provides higher incentives to specialize.

When specialization costs are pecuniary, an investor may be able to limit specialization by limiting the amount of financing. This is only efficient when the entrepreneur over specializes, i.e., when the effect of specialization on liquidation value is more important than its effect on long term productivity. A similar argument applies to the use of costly contractual clauses which limit specialization through verifiable characteristics of the assets.\(^2\)

There is a recent body of empirical work analyzing the relationship between asset specialization and financial contracting. This literature finds that redeployable assets receive larger loans with longer maturities and durations (Benmelech et al., 2005), lower the cost of external financing and increase debt capacity (Benmelech and Bergman, 2008), make leasing more likely (Gavazza, 2010), and play a role in investment-cash flow sensitivities by affecting firms’ financing constraints (Almeida and Campello, 2007). Consistent with much of the evidence, our model predicts that specialization, through its effect on liquidation values, should make financing more difficult.

However, we also find that specialization eases financing constraints through improvements in productivity which increase entrepreneurs’ incentive to repay. Therefore, a unique and untested prediction of our model is that the overall impact of specialization on the terms of financial contracting and financing capacity may sometimes be positive. For example, an airline could specialize on one specific type of aircraft and interior design, which may have lower liquidation values in the case of bankruptcy (Gavazza, 2010). However, the airline could nevertheless get better financing terms if specialization improves productivity by minimizing maintenance, inventories of spare parts, and pilot training costs (see Boguslaski, Ito

\(^2\)Smith and Warner (1979) find that extensive direct restrictions on production/investment policy would be expensive to employ and are not often observed among bond covenants.
and Lee, 2004, for a study of Southwest Airlines).

Our paper is closely related to the general literature on incomplete financial contracting developed by Hart and Moore (1994, 1998) and Bolton and Scharfstein (1990), among others. The main difference is the addition of a specialized investment decision which affects the project’s payoffs and, as a consequence, the terms of contracting. This addition also brings us close to the literature that analyzes the principal agent problem under incomplete contracts where investment is made by the agent but the ownership of the end product or asset is generally more efficient under the principal. In this literature, the agent may not have efficient incentives to invest if the principal can hold him up ex-post. Moreover, there could be a countervailing effect if the agent’s investment also increases his outside option, or reduces that of the principal. In such a setting, contract renegotiation and sharing of surplus plays a central role (see, e.g., MacLeod and Malcomson, 1993, Noldeke and Schmidt, 1998, Che and Hausch, 1999, Edlin and Hermalin, 2000). In contrast, in our model the ownership of the asset is always more efficient under the entrepreneur and the investor plays no role other than to provide financing, making the assumption that financing is provided competitively reasonable. Here, the inherent problem relates to the entrepreneur’s inability to pledge the cash flows from his project. Consequently, the goal of contracting is to maximize the amount of cash the entrepreneur can credibly pledge to the investor in order to minimize the amount of liquidation. The first best is generally not achievable since allocating liquidation rights to the investor is incentive compatible for the entrepreneur and assets must be liquidated in some states of the world to ensure feasibility. Renegotiation plays little role here because the optimal contract we derive achieves efficient allocation of ownership and cash ex-post.

We assume that liquidation is inefficient, but this need not always be the case (Zender, 1991; Aghion and Bolton, 1992; Berkovitch, Israel and Zender, 1997), especially when investors are specialized in redeploying assets (Habib and Johnsen, 1999). Therefore, our
results are more likely to apply to financing provided by general financiers such as banks rather than by financiers specialized in redeploying assets.

2 The Model

An entrepreneur (EN) is endowed with a two-period project which requires an initial investment of $I$ and returns a cash flow $C_t$ in each of the subsequent periods, $t = 1, 2$. These cash flows are observable by the entrepreneur but not by the investor or the courts and therefore cannot be contracted upon.\(^3\) The entrepreneur has wealth of $W \leq I$ and needs to raise the remaining amount $Z \equiv I - W$ from an investor. The first period cash flow $C_1$ is random and is distributed according to the density function $f(\cdot)$ with support in $[C_1, \overline{C}_1]$, where $C_1 \geq 0$ and $\overline{C}_1 \leq \infty$.

The entrepreneur has some flexibility in how to invest the capital for his project and may choose the level of specialization once he has obtained financing, i.e., specialization is not contractible, although it is observed by investors. Specialization, denoted by $k \geq 0$, is privately costly for the entrepreneur, with the cost given by $g(k)$, which is increasing and convex, with $g'(0) = 0$. This is equivalent to assuming that the cost $g(\cdot)$ of specializing the assets is non-pecuniary and, for instance, represents time and effort spent by the entrepreneur in modifying more standard assets to his specific requirements. It also captures an entrepreneur’s investment in human capital. In Section 4.1, however, we extend the analysis to the case where the cost of specialization is pecuniary and needs to be paid for at the same time that the investment is made.

The long term return from the project, $C_2$, is increasing and concave in $k$: $\frac{\partial C_2}{\partial k} > 0$ and $\frac{\partial^2 C_2}{\partial k^2} \leq 0$. However, the value of assets under alternative use decreases with specialization: the investment’s liquidation value at $t = 1$ is equal to $L(k) < I$ for all $k \geq 0$, and is decreasing

\(^3\)This assumption rules out the type of contracts studied in Harris and Raviv (1995).
and convex in $k$: $\frac{\partial L}{\partial k} \leq 0$ and $\frac{\partial^2 L}{\partial k^2} \geq 0$. The liquidation value of the assets decreases over time, and for simplicity we assume that it is equal to zero at time 2.

We assume that $E[C_1] + L(0) > I$, which implies that the project could be financed by giving the investor liquidation rights, as long as the assets have not been specialized and the entrepreneur could commit to pay out all first period cash flows. We also assume that $C_2 \geq L$ for all $k$. This allows us to focus on the case where the entrepreneur is reluctant to relinquish control since continuing the project is privately and socially preferable to liquidating the firm’s assets. The market for financing is competitive, with investors offering contracts to the entrepreneur at time 0, and the entrepreneur choosing the best contract.

Finally, we follow Bolton and Scharfstein (1990, 1996) and Faure-Grimaud (2000) and assume that the financial contracts offered by investors can be composed of a variable $P(R)$ that represents a payment made by the entrepreneur to the investor when the entrepreneur makes a report $R$. They also contain a second component, $\beta(R)$, which represents the probability that the project is allowed to continue into time 2 rather than being terminated at time 1 and having the investor receive the proceeds from liquidation $L$. We can equally well interpret $1 - \beta$ as the fraction of the assets that get liquidated, with the project then yielding $\beta C_2$ at time 2 from the remaining assets.

### 3 Optimal Financial Contract

We solve the model by backward induction. Suppose that a contract $\{P, \beta\}$ has been accepted by the entrepreneur in exchange for receiving $Z$ from some investor. The entrepreneur solves the following problem to find the optimal amount of specialization:

$$\max_k \pi_k^{EN} = E[C_1 - P + \beta C_2(k)] - g(k),$$

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4We note that any feasible contract $\{P, \beta\}$ described here can be implemented by a truth-telling direct mechanism (see, e.g., Myerson, 1979) where the reports $R$ are simply statements about $C_1$. In what follows we therefore use direct mechanisms to characterize the optimal contract.
where the expectation is taken with respect to the first period cash flow, $C_1$. The first order condition for (1) is

$$E [\beta] \frac{\partial C_2}{\partial k} - \frac{\partial g}{\partial k} = 0,$$  \hspace{1cm} (2)

and we define $k^{NC}$ as the solution to (2), where $NC$ stands for “no commitment” to a specialization level. Since $g$ is convex with $g' = 0$ for $k = 0$, we immediately have that $k^{NC} > 0$, so that the entrepreneur will find it optimal to specialize the project’s assets.

Next we characterize the optimal financial contract. Since it is always better to continue the project, i.e., $C_2 > L$, the goal is to design a contract that maximizes the continuation probability but that satisfies the participation and incentive compatibility constraints of all parties. The following result characterizes the equilibrium financial contract. The result is a straightforward application of Faure-Grimaud (2000), and we therefore omit the proof.

**Proposition 1.** The optimal financial contract is characterized by a cutoff value $\hat{C} = C_2 \left(k^{NC}\right) + T$ of the first period cash flow such that (i) for $C_1 < \hat{C}$, $P = C_1$ and $\beta = \frac{C_1 - T}{C_2} < 1$ for some constant $T \leq 0$; and (ii) for $C_1 \geq \hat{C}$, $P = \hat{C}$ and $\beta = 1$. The constant $T$ is chosen as the lowest value such that $\hat{C}$ satisfies the investor’s participation constraint with equality:

$$E \left[P|k^{NC}\right] + E \left[1 - \beta|k^{NC}\right] L(k^{NC}) - Z = 0.$$  \hspace{1cm} (3)

The optimal financial contract resembles a debt contract. The entrepreneur promises to pay a fixed amount $\hat{C}$. When the full payment cannot be made, all the first period cash flow accrues to the investor, plus the investor is given the right to liquidate the assets with probability $1 - \beta$. This is similar to the optimal contract found in Bolton and Scharfstein (1990, 1996), as well as in Hart and Moore (1998). One interpretation of this optimal contract from an empirical point of view is a debt contract with deviations from the absolute priority rule (for a discussion of this, see Faure-Grimaud, 2000).
Specializing the assets is beneficial to the entrepreneur since it increases his long term return $C_2$. However, once the optimal financial contract is in place, specialization comes at the expense of the investor. To see this, note that the payoff to the investor is given by

$$\int_{L_1}^{C_1} (P(R) + (1 - \beta(R)) L(k)) f(R) dR - Z.$$  \hspace{1cm} (4)

Given that the contract is fixed, the payoff to the investor, (4), is reduced as a result of specialization whenever $\frac{\partial L}{\partial k} < 0$.

Finally, we note that since the contract $\{P, \beta\}$ is designed to induce truth-telling by the entrepreneur, i.e., the payment $P$ and the probability of continuation $\beta$ are structured so that it is incentive compatible for the entrepreneur to announce the realized first period cash flow truthfully, the contract is ex post renegotiation proof: there is nothing either party could offer that would make both of them better off and lead to a Pareto improvement.

### 3.1 Specialization and Feasibility

The analysis above shows that, ceteris paribus, an investor’s ability to recover his investment increases with the liquidation value $L$ of the assets. Given that specialization decreases $L$, it would seem that specialization must reduce the likelihood of obtaining financing. However, it is also true that the entrepreneur’s willingness to repay the investor increases with the second period cash flow $C_2$. Since $C_2$ increases with specialization, this creates a countervailing force that can make financing more likely as a result of specialization.

To study whether specialization tightens or loosens financing constraints, we define a firm’s financing capacity as the maximum amount of external financing that can be obtained while satisfying the investor’s participation constraint. This happens when the variable $T$ from Proposition 1 is equal to zero, as $-T \geq 0$ represents the amount of slack in the participation constraint of the investor since, if $-T > 0$, an entrepreneur could raise additional financing by increasing the probability of liquidation $1 - \beta$ and increasing the cash payment.
Define $C_{NC}^2 = C_2(k_{NC})$, $L_{NC}^{} = L(k_{NC})$ and the financing capacity by $D$, which is simply the maximum expected payoff for an investor and is obtained by choosing $T = 0$ in (4), gross of the amount of financing provided ($Z$). $D$ can be written as

$$D_{NC} = \int_{C_1}^{C_{NC}^2} C f(C) dC + C_{NC}^2 \int_{C_1}^{C_1} f(C) dC + \int_{C_1}^{C_{NC}^2} \frac{C_{NC}^2 - C}{C_{NC}^2} L_{NC}^{} f(C) dC$$

(5)

for the case where the entrepreneur chooses a level of specialization $k_{NC}^{}$.

To study the effect of specialization on feasibility, we compare the financing capacity of a project where specialization is determined endogenously with that for a project that has no scope for specialization, so that $k = 0$. As above, define $C_0^2 = C_2(0)$ and $L_0^{} = L(0)$.

To obtain the financing capacity for the case of no specialization, $D_0^{}$, we replace $C_{NC}^2$ and $L_{NC}^{}$ with $C_0^2$ and $L_0^{}$ in (5). Whether specialization increases or decreases financing capacity depends on its relative impact on the liquidation value of assets and long term cash flows.

To show that specialization can tighten financing constraints, consider the rather extreme case where $L(0) > 0$ and $L(k) = 0$ for $k > 0$. In other words, any specialization destroys the entire liquidation value. Since $k_{NC}^{} > 0$, as a consequence $C_{NC}^2 > C_0^2$. Now consider $D_{NC} - D_0^{}$, which from (5) is

$$D_{NC} - D_0^{} = \int_{C_0^2}^{C_{NC}^2} (C - C_0^2) f(C) dC + \int_{C_1}^{C_1} (C_{NC}^2 - C_0^2) f(C) dC + \int_{C_0^2}^{C_{NC}^2} \frac{C_{NC}^2 - C}{C_{NC}^2} L_{NC}^{} f(C) dC$$

(6)

Since $L_{NC}^{} = 0$, (6) simplifies to $D_{NC} - D_0^{} =$

$$\int_{C_0^2}^{C_{NC}^2} (C - C_0^2) f(C) dC + (C_{NC}^2 - C_0^2) \int_{C_1}^{C_1} f(C) dC - \int_{C_0^2}^{C_{NC}^2} \frac{C_0^2 - C}{C_0^2} L_0^{} f(C) dC.$$  (7)

The last term of (7) is strictly negative. The first two terms are positive and increase in $C_{NC}^2$, which reflects that specialization may increase financing capacity through its impact on second period cash flows. However, $D_{NC} - D_0^{}$ will be negative if the increase in continuation
value, $C_2$, is sufficiently small. More formally, if $C_2^{NC} - C_2^0 \leq \delta$, for $\delta$ sufficiently small we can see from (7) that $D^{NC} - D^0 < 0$, so that specialization reduces financing capacity. This finding reflects the more commonly stated argument that asset specificity makes debt financing more difficult since it reduces the “collateral” value (i.e., the redeployability) of assets (see Williamson, 1988, for a discussion). While we illustrate this effect for the extreme case where all liquidation value is destroyed when there is any amount of specialization, it can readily be seen to occur when the negative effect of specialization on the liquidation value of assets is large relative to its positive effect on long term cash flows.

We can also establish the opposite case, that specialization may loosen financing constraints. To see this, take the extreme case where $\frac{\partial L}{\partial k} = 0$ for all $k$, so that specialization has no effect on liquidation value. Now consider again $D^{NC} - D^0$, from (6):

$$D^{NC} - D^0 = \int_{C_2^0}^{C_2^{NC}} (C - C_2^0) f(C) dC + \int_{C_2^{NC}}^{C_1} (C_2^{NC} - C_2^0) f(C) dC$$

$$+ \int_{C_1}^{C_2} \frac{C (C_2^{NC} - C_2^0)}{C_2^{NC} C_2^0} L^0 f(C) dC + \int_{C_2^0}^{C_1} \frac{C_2^{NC} - C}{C_2^{NC} C_2^0} L^0 f(C) dC,$$

since $L(k) = L^0$ for all $k$. Each of the terms in (8) is positive, so that clearly $D^{NC} - D^0 > 0$, and specialization increases financing capacity. When $L(k) = L^0$ for all $k$, one can also show that specialization improves the terms of financial contracting. A formal derivation can be found in the proof of Lemma 4.

To the best of our knowledge, this result has not been analyzed before. Moreover, while we again illustrate the effect for the extreme case where specialization has no effect on liquidation value and only increases the continuation value, from (6) it can be seen to arise whenever the reduction in liquidation value resulting from specialization is small relative to the increase in long term cash flows, so that the increased payments that can be pledged out of first period cash flows have a larger impact on financing capacity.
3.2 Outside Financing and Specialization

The need for outside financing introduces two inefficiencies. First, even though it is optimal to continue the project, in some states the creditor liquidates the assets. Second, liquidation prevents the entrepreneur from collecting the fruits of his investment in specialized assets.

**Lemma 1.** When the agent cannot commit how much to specialize, the optimal level of specialization ($k^{NC}$) with outside financing is always lower than the first best level of specialization ($k^{FB}$).

The intuition for the result in Lemma 1 is simple: outside financing introduces the possibility of liquidation, in which case the entrepreneur cannot benefit from his investment in specialization. As a result, the entrepreneur underinvests in specialization compared to the first best. This is reminiscent of the debt overhang problem that has been widely studied in finance. However, different from the classic debt overhang problem, here the entrepreneur does not share the return on his investment (i.e., specialization) with the investor. Rather, it is the friction introduced by the nonverifiability of cash flows that forces him to allocate control rights to the investor, giving the investor the right to terminate the project.

To obtain a measure of the extent of the inefficiency, here we follow an approach similar in spirit to that in the dynamic contracting literature (see, e.g., Albuquerque and Hopenhayn, 2004, Clementi and Hopenhayn, 2006, or DeMarzo and Fishman, 2007) and consider how the entrepreneurial value is affected by the degree of inefficiency from outside financing. For a given financing need $Z$, the entrepreneur’s value from the project, assuming an optimal financial contract $(P, \beta)$, is given by (1). In equilibrium, the investor’s participation constraint, given by (3), must be satisfied with equality. Therefore, we can solve (3) for the expected cash payment to the investor, $E[P]$, and substitute into (1) to get

$$V^{EN}(Z) = E[C_1] + E[\beta(Z)]C_2(k) + E[1-\beta(Z)]L(k) - g(k) - Z.$$ (9)
$V^{EN}(Z)$ represents the value of the project for the entrepreneur for a given level of outside financing $Z$. It is worthwhile noting from (9) that part of the entrepreneur’s value comes from the liquidation value $L$ of the assets. While this source of value is not internalized by the entrepreneur since he chooses how much to specialize after the financial contract is in place, it nevertheless represents an equilibrium relationship.

Define now $V^{FB}(Z)$ as the value the entrepreneur would obtain if there were no financial frictions but an amount $Z$ of external financing were still required:

$$V^{FB}(Z) = E[C_1] + C_2(k) - g(k) - Z. \quad (10)$$

One measure of the inefficiency associated with outside financing can now be obtained from the ratio $\frac{V^{EN}(Z)}{V^{FB}(Z)}$. This inefficiency is directly linked to amount of outside financing and leverage. To see this, define $V(Z) = V^{EN} + Z$ as the overall value of the project. The leverage ratio is then $\frac{Z}{V(Z)}$, and we can state the following result.

**Lemma 2.** As the amount of outside financing $Z$ increases, leverage $\frac{Z}{V(Z)}$ increases, the inefficiency introduced by outside financing increases, i.e., $\frac{V^{EN}(Z)}{V^{FB}(Z)}$ decreases, and the level of specialization $k^{NC}$ decreases.

As the amount of outside financing increases the probability of liquidation increases, which decreases the entrepreneur’s incentive to specialize, as argued above. As a result, the entrepreneur’s value $V^{EN}(Z)$ decreases both because he faces a higher probability of liquidation and because of the lower investment in specialization.

### 3.3 The Value of Commitment

Here, we study the implications of the entrepreneur’s inability to commit to a particular level of specialization by comparing the solution obtained above, $k^{NC}$, to what obtains if the entrepreneur could commit to a level of specialization before obtaining financing.
Solving by backward induction for the case when the entrepreneur can commit, it is straightforward to see that for any \( k \), the optimal contract will be given by the solution in Proposition 1. However, the terms \( P \) and \( \beta \) of the contract now depend directly on the degree of specialization \( k \). Assuming that the investor’s participation constraint,

\[
E[P(C_1)] + E[1 - \beta(C_1)]L(k) - Z \geq 0,
\]

(11)
can be satisfied with equality\(^5\), we can solve for \( E[P] \) and substitute into the entrepreneur’s objective function, (1), and take the FOC to obtain the optimal level of specialization \( k^C \)

\[
\frac{\partial E[\beta]}{\partial k} (C_2(k) - L(k)) + \frac{\partial L}{\partial k} (1 - E[\beta]) + E[\beta] \frac{\partial C_2}{\partial k} - \frac{\partial g}{\partial k} = 0.
\]

(12)

Under commitment, the entrepreneur can always choose the same level of specialization that would be chosen under no commitment, \( k^{NC} \). If the level of specialization that is actually chosen under commitment is different from \( k^{NC} \), it must be that the entrepreneur’s payoff is higher. We summarize this argument in the following preliminary result.

**Lemma 3.** The entrepreneur is at least as well off (i.e., \( \pi^{EN} \) is higher) when he can commit to how much he will specialize the firm’s assets.

While Lemma 3 relates to the overall payoff of the entrepreneur, it does not speak to the question of how being able to commit would affect the entrepreneur’s specialization decision. Since the entrepreneur would internalize the effect of his specialization decision if he could commit, intuition suggests that the effect of specialization on long term cash flows relative to liquidation value will determine whether he would specialize more or less.

\(^5\)If the participation constraint of the investor is not satisfied at \( k^C \), finding the solution for the optimal amount of specialization requires including the participation constraint as an additional constraint that may be binding. This can either raise the optimal \( k \) or lower it, depending on whether greater specialization increases or reduces financing capacity.
Lemma 4. (1) Suppose that $L(k) = L(0)$ for all $k$. Then $0 < k^{NC} < k^{C}$. (2) Assume that $\frac{\partial L}{\partial k} < 0$ for all $k \geq 0$. Then there exists $\epsilon$ small enough such that, if $\frac{\partial C_2}{\partial k} \bigg|_{k=0} \leq \epsilon$, $k^{NC} > k^{C} > 0$.

Part (1) of Lemma 4 establishes that, whenever specialization has no effect on asset liquidation value, the entrepreneur’s inability to commit leads to a lower level of specialization than if he could commit. The intuition stems from recognizing that specialization, by increasing long term cash flows, improves the entrepreneur’s ability to pledge to pay out of the first period cash flows. In equilibrium, this improves the terms of contracting and leads to greater specialization when the entrepreneur can commit, since he then internalizes the effect of specialization on the terms of contracting. Moreover, since the optimal degree of specialization is continuous in the liquidation value, the result can be shown to hold more generally even when specialization reduces liquidation value, as long as this reduction is small relative to the increase in continuation value.

Part (2) of Lemma 4 establishes the opposite result, namely that when specialization has a relatively small effect on long term cash flows the entrepreneur will find it optimal to specialize the assets more when he cannot commit than when he can. The intuition is similar in that, when specialization has little benefit to the entrepreneur but, all things equal, reduces the payoff to the investor, the entrepreneur will internalize the worsening of the terms of contracting when he can commit and will therefore choose to specialize less.

As a final point, the arguments above are predicated on the solutions to both the commitment and the no commitment cases being defined by the two respective first order conditions. It is readily seen that many such cases exist, and we provide an explicit example below.
3.4 A Numerical Example

We illustrate when specialization improves financing capacity and how the level of specialization may differ if the entrepreneur could commit with an example. Assume that the liquidation value is equal to \( L = \frac{5}{(1+k)} \) so that, as \( \omega \) increases, the negative impact of specialization on the liquidation value increases. The first period cash flows \( C_1 \) are uniformly distributed between \( C_1 = 0 \) and \( C_1 = 20 \), and the second period cash flows are given by \( C_2 = 10 + \sqrt{k} \). The cost of specialization is \( g(k) = \frac{1}{2}k^2 \).

Figure 1 illustrates how financing capacity \( D \) changes as we vary \( \omega \) between \([0, 2]\) for the case where the agent cannot commit and the degree of specialization is chosen optimally at \( k^{NC} \). The horizontal line, at \( D^0 = 8.75 \), represents the case where there is no specialization, so \( k = 0 \). For the case where specialization is optimally chosen we see that for low values of \( \omega \) financing capacity, \( D^{NC} \), is higher than with no specialization. The reason is that, when \( \omega \) is low, specialization has a small impact on liquidation value relative to its effect on second period cash flows. However, for larger value of \( \omega \), financing capacity is lower under specialization because the effect of specialization on the liquidation value is large, and at some point this becomes a more important effect than the increase in \( C_2 \).

Figure 2 illustrates how the optimal level of specialization changes as we vary \( \omega \) for the no-commitment, commitment, and first-best cases. We assume that the entrepreneur borrows 7 to finance the project, which is feasible in all cases we analyze. As above, when \( \omega \) is low the negative impact of specialization on the liquidation value is dominated by its positive impact on long term cash flows, which improves the terms of the financial contract. As a result, if the entrepreneur could commit he would internalize this improvement in the terms of contracting and would choose a higher level of specialization compared to what he chooses when he cannot commit (as argued in part 1 of Lemma 4). However, as \( \omega \) increases the effect of specialization on the liquidation value again dominates. Since this worsens the
terms of contracting, under commitment the entrepreneur specializes less compared to when he cannot commit (as argued in part 2 of Lemma 4).

4 Model Extensions

4.1 Pecuniary Cost of Investment in Specialization

The assumption that the cost of specializing the assets, \( g(k) \), is non-pecuniary and privately borne by the entrepreneur fits well such investments as those in human capital, which are likely to be privately costly to the entrepreneur. However, there are also specialized investments whose costs are pecuniary. In this case specializing means having to purchase assets that are specifically tailored for the project and are thus more costly than “standard” assets.

We study this issue in a setting where the project’s scale \( I \) can also be chosen by the entrepreneur. We do this primarily in order to prevent the investor from being able to dictate the level of specialization through the size of the investment, since it gives the entrepreneur some discretion on how to allocate funds between specialization and project scale. Specifically, we modify the setting slightly and assume that when the entrepreneur chooses \( I \), the project produces cash flow \( C_1 \) in the first period, as before. It then produces \( C_2(k)H(I) \) in the second period, where \( H(I) \) is an increasing and concave function of \( I \), with \( H' \to 0 \) as \( I \to \infty \), which implies decreasing returns to scale in investment. The project has liquidation value \( L(k)I \) at time 1 such that \( L(0) < 1 \), and equals 0 at time 2. The cost of specialization, \( g(k) \), must be paid for at the time the assets are purchased. We assume that \( C_2(k)H(I) > L(k)I \) for \( I < \bar{I} \), where \( \bar{I} \) is “large,” so that liquidation is inefficient. For simplicity, we assume that the entrepreneur’s wealth \( W \) is zero. We define \( A \) as the total amount of financing that is obtained, so that \( A \geq I + g(k) \).

\( ^6 \) We study the case where the investor can limit the amount of specialization by limiting the amount of financing in Section 4.3.
The timing is as follows. First, investors offer contracts \( \{ A, P, \beta \} \), where \( P \) and \( \beta \) are the repayment function and the probability of continuation, respectively, as before. Then, the entrepreneur chooses \( I \) and \( k \), and \( C_1 \) and \( C_2 \) are realized, as before. We solve by backward induction. Given a contract \( \{ A, P, \beta \} \), the entrepreneur maximizes

\[
\max_{I,k} \pi^{EN} = E[C_1 - P(R) + \beta(R)C_2(k)H(I)] - g(k).
\] (13)

Assuming that the entrepreneur does not borrow more than he needs for investment, his budget constraint is \( I + g(k) = A \). Solving for \( I = A - g(k) \), we can write (13) as

\[
\max_k \pi^{EN} = E[C_1 - P(C_1) + \beta(C_1)C_2(k)H(A - g(k))] - g(k),
\] (14)

which is a maximization problem with respect to \( k \) only. The first order condition is

\[
\frac{\partial \pi^{EN}}{\partial k} = E[\beta] \left( \frac{\partial C_2}{\partial k} H - C_2 \frac{\partial H}{\partial I} \frac{\partial I}{\partial g} \frac{\partial g}{\partial k} \right) - \frac{\partial g}{\partial k} = 0.
\] (15)

The first term is positive and concave in \( k \) while the second term is always negative and convex in \( k \). A solution \( k^{PC} \) to (15) exists such that \( k^{PC} > 0 \). The equilibrium level of specialization balances the improved long term cash flows due to specialization with a smaller project scale and higher cost of specialization.

For a given level of financing \( A \), it is straightforward to see that the optimal contract \( \{ P, \beta \} \) is qualitatively the same as before\(^7\). To understand the effect of pecuniary specialization costs, we compare this case to the case where specialization costs are non-pecuniary, as well as to the first best. For the latter, it is straightforward to see that, since \( E[\beta^{PC}] < 1 \), from (15) there is an inefficiently low level of specialization relative to when continuation always takes place.

\(^7\)The entrepreneur’s optimal choice of external financing \( A \) can be solved using the first order condition, which yields that the entrepreneur increases the project’s scale taking into account that increases in scale have a diminishing impact on long term cash flows, which must be balanced against the marginal cost of the investment that must be financed.
To compare the pecuniary cost case to the case where specialization costs are non-pecuniary, we assume that in the non-pecuniary cost case the second period cash flows are given by $C_2(k)H(I)$, the liquidation value is given by $L(k)I$, and the scale of investment is of the same size across both cases and equal to $I = I^{PC}$, where $I^{PC} = A - g(k^{PC})$. This comparison is relevant as it implies that the cash flows produced by the project in both scenarios would be the same if the entrepreneur were to choose the same level of specialization.\footnote{An alternative comparison would be to the case where the total amount of external financing is constant across both scenarios and equal to $A^{PC} = I^{PC} + g(k^{PC})$, which provides similar results.}

Note now that, comparing the pecuniary to the non-pecuniary cost case, the second period cash flows will differ only as a result of the specialization decision. Since the investor provides a greater amount of financing when specialization costs are pecuniary, the required expected payment $E[P^{PC}]$ and liquidation probability $E[1 - \beta^{PC}]$ will need to be higher for every anticipated level of specialization (Lemma 2). This implies that the entrepreneur will optimally choose a lower level of specialization when these costs are pecuniary, as the probability of continuation will be lower.

4.2 Delaying Investment and Specialization

In our model the investor’s liquidation rights give him some control over whether the firm can proceed to the second period. However, if the investment is partially delayed the investor can also exert some control on whether the entrepreneur receives financing for the second period. This may be beneficial for the entrepreneur if delayed investment improves the terms of contracting. In this section, we introduce the possibility that financing and investment can be done in stages and explore its interaction with the specialization decision.

Assume that the project is feasible when all of $I$ is invested at $t = 0$ and that the entrepreneur has no wealth. The entrepreneur can choose to obtain only $\phi I$ in financing and to invest a fraction $\phi \in [0, 1]$ of the total investment $I$ at $t = 0$, in which case the first
period cash flow will be \( \rho(\phi)C_1 \), where \( \rho(\phi) \) is a continuous function in \([0,1]\), with \( \rho' \geq 0 \), \( \rho(0) = 0 \), and \( \rho(1) = 1 \). The function \( \rho \) captures the fact that investing anything less than the full amount initially (i.e., if \( \phi < 1 \)) may lead to a reduction in first period cash flows. After \( \rho(\phi)C_1 \) is realized and the entrepreneur reports \( R \), the investor receives payment \( P(R) \) and provides financing for the second stage investment with probability \( \beta(R) \).

To realize the second period cash flows, \( C_2 \), full investment is required (i.e., \( C_2 = 0 \) if \( (1 - \phi)I \) is not invested at time 1). If the project is not continued, the assets from the initial investment are liquidated. We assume that the investment’s liquidation value at \( t = 1 \) is proportional to the time zero investment and equal to \( \phi L \), where \( L < I \) for all \( k \geq 0 \), and \( \partial L / \partial k \leq 0 \), as usual. There is no liquidation value at time 2. The rest of the model remains the same.

Since the initial amount of financing provided is now less than the full amount \( I \), the financial contract must include the additional term reflecting the degree of delay in investing, \( \phi \), but is otherwise similar to what we obtained before. Given a contract \((P, \beta, \phi)\), the payoff to the entrepreneur is now

\[
\pi^{EN}(R; C_1) = E[\rho(\phi)C_1 - P(R) + \beta(R)C_2(k)] - g(k). \tag{16}
\]

The expected payoff of the investor is

\[
\int_{C_1}^{C_1} (P(C) - \beta(C)(1 - \phi)I + (1 - \beta(C))\phi L) f(C) dC - \phi I \geq 0. \tag{17}
\]

When full investment is delayed, the first period cash flow \( \rho(\phi)C_1 \) and the liquidation value \( \phi L \) decrease compared to the case of full investment. On the other hand, the initial investment is reduced to \( \phi I \), which decreases the loss to the investor when \( C_1 \) is low and he is not fully repaid. While the first two effects reduce the continuation probability, the third effect increases it. The optimal amount of delay depends on the overall effect of these

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\(^9\)This is as in Bolton and Scharfstein (1990), who assume that the financial contract stipulates a commitment to provide continued financing as long as the initial repayment is made.
factors on the entrepreneur’s payoff. It is easy to see that it is never optimal to defer all investment until time 1 since with no initial investment there will also be no cash flow in the first period, as well as no assets to repay an investor. The payoff of the entrepreneur would be zero, which is strictly less than when $\phi = 1$ given that financing is assumed to be feasible under full investment. However, we can show that there is a broad class of cases where the entrepreneur optimally chooses to delay investment, so that $\phi^* \in (0, 1)$.

**Proposition 2.** $\phi \in (0, 1)$ is optimal if $\lim_{\phi \to 1} \rho' = 0$.

The reason deferral of investment is optimal derives from the fact that when $\lim_{\phi \to 1} \rho' = 0$, the reduction in first period cash flows is of second order as $\phi \to 1$. Deferral of investment does have a first order impact on the liquidation value $\phi L$. However, in default states, where the reduction of liquidation value matters, the initial investment is also reduced in the same proportion, $\phi I$. Overall, therefore, the investor’s loss in liquidation states is equal to $\phi (I - L)$, which increases with $\phi$. As a result, (at least some) delay improves the terms of contracting and increases the entrepreneur’s payoff when it has little impact on first period cash flows.

As the next result shows, deferral is only optimal when its effect on the continuation probability is positive. This has implications for the optimal degree of specialization.

**Lemma 5.** If it is optimal to delay the investment, so that $\phi < 1$, then the continuation probability, $E [\beta]$, is greater than under full investment (i.e., $\phi = 1$). Moreover, the entrepreneur chooses to specialize the assets more (i.e., larger $k$) compared to the full investment case.

The lemma establishes that projects with staged investments should have better financial contract terms - specifically, they should be allowed to continue more often - and greater specialization compared to similar projects where full investment is made from the beginning.
4.3 Controlling Specialization

As discussed above, the inability to commit to a level of specialization introduces various inefficiencies, including inefficient liquidation, infeasibility of obtaining financing, and over/under specialization. Therefore, one might expect that the investor and the entrepreneur would agree, if possible, to circumvent the problems introduced by the entrepreneur’s inability to commit. In this section we analyze some ways in which the commitment problem could be solved, at least partially.

We consider the case where the investment in specialization is pecuniary but the size of the project is not scalable. Assuming the entrepreneur has no wealth, i.e., $W = 0$, the investor can limit the amount of specialization by providing only $A = I + g(k)$, where $k$ is whatever amount of specialization deemed optimal by the investor. Given that the market for financing is competitive and the payoff to the investor is zero in equilibrium, $k$ should be set so as to maximize the entrepreneur’s payoff. The solution for this case will be similar to the case when the entrepreneur cannot commit to a certain level of specialization, except that now there is an upper limit imposed on specialization. We assume that if any of the funds raised are not used, they remain with the entrepreneur, which introduces the funds raised into his payoff function:

$$\max_k \pi^{EN} = E[C_1 - P + \beta C_2(k) + I + g(k) - g(k)],$$  \hspace{1cm} (18)$$

subject to $k \leq \bar{k}$.

Given that $g(\bar{k})$ is constant, the first order condition with respect to $k$ is exactly the same as in the base case scenario. The profit with outside financing is maximized when the entrepreneur chooses a level of specialization equal to $k^C$, the specialization level when the entrepreneur can commit. Therefore, if $k^C < k^{NC}$, the investor can choose $\bar{k} = k^C$ and ensure that the entrepreneur chooses the level of specialization obtained in the commitment case,
which is (second best) socially optimal. However, if $k^C > k^{NC}$, then introducing an upper bound does not solve the problem stemming from the entrepreneur’s inability to commit since the optimal amount of specialization is $k^C$ but the entrepreneur will always want to choose a lower level equal to $k^{NC}$.

**Lemma 6.** If $k^C < k^{NC}$, then the investor can limit the level of specialization to $k^C$ by offering to finance $A = I + g(k^C)$. Otherwise, if $k^C > k^{NC}$, limiting the amount of financing does not improve the entrepreneur’s payoff.

The analysis above assumes that the project is feasible at both the commitment and the no-commitment levels of specialization. If the project is not feasible at the no-commitment level of specialization but would be feasible at a lower of level, then placing an upper bound on specialization could be helpful as well for obtaining financing.

Although asset specificity may not be verifiable, certain characteristics of the investment could be observed and costly contractual clauses could then be used to limit an entrepreneur’s ability to specialize the assets. As in the analysis above, such clauses would be beneficial only when the entrepreneur over-specializes relative to what he would do if he could commit.

## 5 Comparative Statics and Empirical Predictions

Prior literature finds that redeployable assets receive larger loans with longer maturities and durations (Benmelech et al., 2005), lower the cost of external financing and increase debt capacity (Benmelech and Bergman, 2008), make leasing more likely (Gavazza, 2010), and play a role in investment-cash flow sensitivities by affecting firms’ financing constraints (Almeida and Campello, 2007). Consistent with the evidence, our model predicts that specialization, through its effect on liquidation values, should make financing more difficult.

However, we also find that specialization eases financing constraints by improving long
term productivity and entrepreneurs’ incentive to pay. Therefore, the overall impact of specialization on the terms of financial contracting and on financing capacity may be positive, in contrast to the prediction that specialization always makes financing more difficult to obtain (Williamson, 1988). Our theory also provides guidance on when specialization is likely to loosen financing constraints. For instance, we expect that specialization in industries where the stock of redeployable assets is low anyway, such as possibly high tech, or where there are few physical assets to begin with, such as service oriented industries (e.g., advertising), would loosen financing constraints. Moreover, even when the value of physical assets is large, the beneficial effect of specialization may nevertheless lead to improvements in financing terms. For example, an airline could purchase only one specific type of aircraft and interior design (like Southwest), which may have lower liquidation values in case of default, especially if other potential buyers use different types of aircraft. However, the airline could nevertheless get better financing terms because using a specific type of aircraft may improve productivity by minimizing maintenance, inventories of spare parts, and pilot training costs (Boguslaski, Ito and Lee, 2004). Identifying industries where there is little cross sectional variation in the effect of specialization on liquidation values but potentially a larger variation in its effect on productivity could also be a way of measuring the positive impact of specialization on the terms of financial contracting.

Testing our predictions requires identifying measures of specialization. Some of the measures used in the literature, like asset tangibility and salability (e.g., Berger et al., 1996; Almeida and Campello, 2007), are based on the potential liquidation values of assets under stress. Although appropriate for their original purpose, they may not be appropriate measures of specialization for the purpose of testing our predictions. Although ideal measures of specialization are context specific, we speculate that investment in firm specific skills by workers, technology, customized machinery and R&D could be potential proxies for spe-
cialization. One could also try to measure the effect of specialization on productivity (see Bartelsman and Dom, 2000, for a review of productivity measures) in addition to liquidation values to identify cases where the overall effect of specialization on financial contracting terms is expected to be positive.

We predict that, ceteris paribus, firms requiring greater outside financing and thus higher leverage should have lower specialization. Similarly, firms with more internal sources of financing should specialize more. This occurs because higher leverage increases the probability of liquidation, preventing the entrepreneur from benefitting from his investment in specialization. Williamson (1988), taking asset specificity as exogenous, predicts that specialized investments are more likely to be financed by equity. Our prediction is slightly different in that we argue that the specialization decision is endogenous and affected by the availability of internal versus external funds. The feedback effect discussed here could be identified by comparing the specialization decision of firms that have similar investment opportunity sets but vary in their availability of internal resources. A somewhat related prediction is that if the cost of specialization is not pecuniary, the entrepreneur specializes more since he does not need to raise external funds to finance specialization.

We also expect staged investment to be associated with projects that have higher levels of specialization. Empirically, staged financing has been mostly studied in the context of venture capital. It has been argued that staging allows venture capitalists to monitor the progress of firms by maintaining the option to abandon projects (Gompers, 1995). Our results imply that a somewhat similar incentive to delay investment may apply more generally and this interacts with the specialization decision, even for projects financed through arm’s length lenders. Although it has been argued that banks engage in staged financing (Stulz, 2000), we are not aware of any study on how it may interact with firms’ specialization decisions.

Finally, our model also predicts that an entrepreneur’s inability to credibly commit to
a particular level of specialization results in over-specialization when specialization has a relatively large effect on liquidation value compared to its effect on productivity. In this case, even if specialization cannot be fully contracted upon, if certain characteristics of the investment can be verified, we expect the entrepreneur to be willing to agree on contractual clauses that limit his ability to specialize assets even when financing is competitive. For instance, aircraft lease agreements often include covenants that limit modifications and alterations that can be done on the aircraft.

6 Conclusion

Investments in specialized assets or firm specific human capital are an important part of economic activity. We study how firms’ specialization decisions interact with financial contracting. We show that the inability to commit to a particular level of specialization affects financial contracting through two channels. First, specialization erodes liquidation value and makes it more difficult to raise outside financing. Second, however, specialization eases financing constraints by improving productivity and entrepreneurs’ incentive to pay. This second effect, to the best of our knowledge, has not been studied before. An entrepreneur’s inability to commit to a level of specialization may result in more or less specialization depending on which of the effects above dominates. This setup provides a rich set of predictions across various forms of specialization.
7 Appendix

Proof of Lemma 1: When self-financing is possible the entrepreneur always continues the project and the first order condition becomes

\[ \frac{\partial C_2}{\partial k} - \frac{\partial g}{\partial k} = 0. \]  

(19)

Comparison of (19) to (2) clearly establishes that, since \( E[\beta] < 1 \) when outside financing is needed, it must be that \( k^{FB} > k^{NC} \), where \( k^{FB} \) satisfies (19). □

Proof of Lemma 2: The investor’s participation constraint, (3), can be expressed as

\[ \int_{C_1}^{\hat{C}} (C + (1 - \beta) L^{NC}) f(C) dC + \hat{C} \left(1 - F(\hat{C})\right) - Z = 0. \]  

(20)

It can be seen that, for every anticipated value of \( k \), when \( Z \) increases the threshold value \( \hat{C} = C_2^{NC} + T \) must increase through an increase in \( T \). This implies that \( E[P] \) increases, and also that \( \beta = \frac{C_1 - T}{C_2} \) will decrease. Since \( \beta \) decreases for every value of \( C_1 < \hat{C} \), a fortiori this implies that \( E[\beta] \) must decrease for every anticipated value of \( k \). In equilibrium, therefore, the optimal level of specialization \( k^{NC} \) must be lower. This also implies that as \( Z \) increases, \( V(Z) = V^{EN} + Z = E[C_1] + E[\beta] C_2 + E[1 - \beta] L - g \) must decrease as well, as otherwise the entrepreneur would prefer a lower value of \( E[\beta] \) even for lower \( Z \). Therefore, leverage \( \frac{Z}{V(Z)} \) strictly increases with the amount of outside financing \( Z \) since the numerator increases and the denominator decreases. Finally, \( \frac{V^{EN}(Z)}{V^{FB}(Z)} \) also decreases since \( V^{EN} < V^{FB} \) and, while both \( V^{EN} \) and \( V^{FB} \) decrease as \( Z \) increases, \( V^{EN} \) decreases more since \( E[C_1] + E[\beta] C_2 + E[1 - \beta] L - g \) decreases as well, as discussed above. □

Proof of Lemma 4: To prove part (1), consider the case when \( L(k) = L(0) \) for all \( k \). We first show that, for this case, \( E[\beta] \) is increasing in \( k \). Suppose that the investor expects \( k \) to increase, so that \( C_2 \) is expected to increase. Suppose now that \( \hat{C} \) remains constant.
or decreases. Given $\beta = \frac{C_1 - T}{C_2}$ for $C_1$ less than the threshold value $\hat{C}$, the slope of $\beta$ will decrease. In that case, since $\beta(\hat{C}) = 1$, $E[\beta]$ would increase, with $E[P]$ staying the same or decreasing. But then the investor’s participation constraint, defined in (20), would no longer be satisfied. Therefore, $\hat{C}$ must increase at the new equilibrium, leading $E[P]$ to increase. Since the participation constraint is satisfied with equality, we must have that $E[\beta]$ increases.

Now, after substituting $P$ from the investor’s participation constraint and eliminating the terms that do not depend on $k$, eqn. (1) is equivalent to

$$\max_k \pi^N = E[\beta](C_2(k) - L(0)) - g(k). \quad (21)$$

We know that $E[\beta^{NC}](C_2^{NC}(k^{NC}) - L(0)) - g(k^{NC}) > E[\beta^{NC}](C_2^{C}(k^{C}) - L(0)) - g(k^{C})$ since $k^{NC}$ maximizes (21) at the equilibrium continuation probability $E[\beta^{NC}]$. Suppose that $k^C < k^{NC}$. Then $C_2(k^C) < C_2(k^{NC})$. Moreover, since specialization has no effect on the liquidation value, we must in fact have $E[\beta^C] < E[\beta^{NC}]$ in order to satisfy the investor’s participation constraint, implying further that $E[\beta^{NC}](C_2^{NC}(k^{NC}) - L(0)) - g(k^{NC}) > E[\beta^C](C_2^{C}(k^{C}) - L(0)) - g(k^{C})$. Choosing $k^C < k^{NC}$ therefore cannot be optimal because under commitment the entrepreneur could choose $k^{NC}$ and do at least as well as in the no commitment case. We therefore must have $k^C \geq k^{NC}$. However, $k^C = k^{NC}$ cannot be optimal for the commitment case since $\frac{\partial E[\beta]}{\partial k} > 0$ if $L' = 0$, so that $k^C = k^{NC}$ would not satisfy the first order condition. This implies that $k^C > k^{NC}$, as desired.

To prove part (2), start with the extreme case where $\frac{\partial C_2}{\partial k} |_{k=0} = 0$. Then, since $C_2$ is concave in $k$ it must be that $\frac{\partial C_2}{\partial k} |_{k>0} = 0$. Since the slope of $\beta$ is not affected by specialization, and the liquidation value goes down, this implies that $\hat{C}$ has to become larger to satisfy the participation constraint, (20). This implies $E[\beta|k>0] < E[\beta|k=0]$. For this case, choosing $k^C = 0$ will be optimal. By continuity, it must be that for $\frac{\partial C_2}{\partial k} |_{k=0} > 0$ but small
enough, choosing \( k^C = 0 \) must still be optimal. However, since \( \frac{\partial C_2}{\partial k} \big|_{k=0} > 0 \) and changes in the liquidation value \( L \) do not directly affect the choice of \( k^{NC} \), the no commitment solution, even in these instances choosing \( k^{NC} > 0 \) will be optimal. Therefore, \( 0 = k^C < k^{NC} \). Note now that, for both the commitment and the no commitment cases, the entrepreneur’s objective function is continuous in the function \( C_2 \), as is the investor’s participation constraint. Therefore, there must exist a function \( C_2(k) \) with \( \frac{\partial C_2}{\partial k} \) large enough for \( k \) in a neighborhood of \( k = 0 \) such that choosing \( 0 < k^C < k^{NC} \) is optimal. □

**Proof of Proposition 2:** Consider first the case where \( \rho(\phi) = 1 \) for \( \phi \geq 1 - \delta \), where \( 0 < \delta < 1 \), in which case \( \rho(\phi) C_1 = C_1 \). Now subtract the participation constraint in the no delay case, (3), from that in the delayed investment case, (17), when the entrepreneur chooses \( \phi = 1 - \delta \), assuming that all parameters including \( T \) and \( k \) remain the same. (3) can be written as

\[
\int_{C_1}^{C_2^{NC} + T} (C + (1 - \beta) L^{NC}) f(C) dC + (C_2^{NC} + T) (1 - F(C_2^{NC} + T)) - I = 0. \tag{22}
\]

With delay, we can write (17) as

\[
\int_{C_1}^{C_2^{NC} + T} (C + (1 - \beta) \phi L^{NC} - \beta(1 - \phi)I) f(C) dC + (C_2^{NC} + T - (1 - \phi)I) (1 - F(C_2^{NC} + T)) - \phi I. \tag{23}
\]

Subtracting (22) from (23) and rearranging yields

\[
\int_{C_1}^{C_2^{NC} + T} (1 - \beta) (I - L^{NC}) (1 - \phi) f(C) dC > 0. \tag{24}
\]

Equation (24) is always positive given that \( I > L \) for all levels of specialization. It also decreases in \( -T \), implying that \( -T \) has to increase to ensure that the investor’s participation constraint binds when the entrepreneur delays investment. If \( -T \) increases, the expected continuation probability \( E[\beta] \) goes up, so that the entrepreneur’s expected payoff is strictly higher when delaying by choosing \( \phi = 1 - \delta < 1 \). Note also that choosing \( \phi = 1 - \delta \) dominates choosing \( \phi = \eta > 0 \) for \( \eta \) small enough since the entrepreneur’s profits go to zero as \( \eta \) goes...
to zero, either because the investor’s participation constraint can no longer be satisfied, or because financing takes place but the probability of continuation goes to zero, whereas the entrepreneur’s profits at $\phi = 1 - \delta$ are strictly positive. Therefore, choosing some delay is optimal for this case.

Consider now the more general case where $\rho(\phi) < 1$ and is increasing in $\phi$ for all $\phi < 1$. If $\lim_{\phi \to 1} \rho' = 0$, then by continuity there is always a value of $\epsilon > 0$ such that, for $\phi = 1 - \epsilon$, the entrepreneur’s expected payoff is higher when delaying, as desired. □

**Proof of Lemma 5:** If delaying is optimal, the entrepreneur’s payoff has to be higher compared to not delaying. Denote the expected repayment and continuation probability in the delay case as $E[C_1] - E[P^\phi]$ and $E[\beta^\phi]$, respectively. If the levels of specialization are the same across both cases, the difference between the payoff with delay and without is equal to

$$\left(\rho E[C_1] - E[P^\phi]\right) - (E[C_1] - E[P]) + (E[\beta^\phi] - E[\beta]) C_2 > 0. \quad (25)$$

Note now that $E[P^\phi] = \rho(\phi) \int_{C_1}^{C\phi} C f(C) dC + \rho(\phi) \int_{\widehat{C}(\phi)}^{C_1} C (\widehat{C}(\phi) f(C) dC$ and $E[P] = \int_{C_1}^{\widehat{C}} C f(C) dC + \int_{\widehat{C}}^{C_1} \widehat{C} f(C) dC$, where we use the notation $\widehat{C}(\phi)$ to indicate the cutoff point with delay. We can replace $E[P^\phi]$ and $E[P]$ into (25) to obtain

$$\rho(\phi) \int_{\widehat{C}(\phi)}^{C_1} (C - \widehat{C}(\phi)) f(C) dC - \int_{\widehat{C}}^{C_1} (C - \widehat{C}) f(C) dC + (E[\beta^\phi] - E[\beta]) C_2 > 0. \quad (26)$$

Contrary to our claim, suppose that $E[\beta^\phi] < E[\beta]$, which can be written explicitly as

$$\int_{C_1}^{C_1 + T(\phi)} \left(\frac{\rho(\phi) C - T(\phi)}{C_2}\right) f(C) dC + \left(1 - F\left(\frac{C_2 + T(\phi)}{\rho(\phi)}\right)\right) \quad (27)$$

$$< \int_{C_1}^{C_1 + T} \left(\frac{C - T}{C_2}\right) f(C) dC + (1 - F(C_2 + T)).$$

Given that the slope of the continuation probability in the delay case, $\frac{\rho(\phi)}{C_2}$, is lower than the slope with no delay, $\frac{1}{C_2}$, it must be that $\widehat{C}(\phi) = \frac{C_2 + T(\phi)}{\rho(\phi)} > C_2 + T = \widehat{C}$, as otherwise $\beta^\phi$ would reach 1 at a lower realization of $C_1$ than $\beta$, which would imply that $E[\beta^\phi] > E[\beta]$, 30
contradicting the assumption. We therefore must have \( \hat{C}(\phi) > \hat{C} \), which implies

\[
\rho E[C_1] - E[P^\phi] - (E[C_1] - E[P]) = \rho(\phi) \int_{\hat{C}(\phi)}^{C_1} (C - \hat{C}(\phi)) f(C) \, dC - \int_{\hat{C}}^{C_1} (C - \hat{C}) f(C) \, dC < 0.
\]

(28)

However, this is not possible since, given the assumption that \( E[\beta^\phi] < E[\beta] \), it would imply that (25) is negative, contradicting the assumption that delaying is optimal. We can also rule out the possibility that \( E[\beta^\phi] = E[\beta] \), which also implies that (25) is less than zero. Therefore, \( E[\beta^\phi] > E[\beta] \), which also implies that the entrepreneur has a higher incentive to specialize the assets when delaying is optimal than if he were unable to delay. □
8 Citations


Boguslaski C., H. Ito, and D. Lee, 2004, “Entry Patterns in the Southwest Airlines Route


Figure 1: The figure plots financing capacity (D) for a project for the case where assets are not specialized (D^0), so that \( k = 0 \), and for the case where the optimal amount of specialization, \( k^{NC} \), is chosen (D^{NC}). For low values of \( \omega \), financing capacity is higher with specialization (D^{NC} > D^0), while for high values of \( \omega \), financing capacity is lower under specialization (D^{NC} < D^0).
Figure 2: The figure plots the optimal degree of specialization under the first best, $k^{FB}$, when no financing is required, for the no commitment case, $k^{NC}$, and for the commitment case, $k^{C}$. For low values of $\omega$, $k^{C} > k^{NC}$ because specialization has little effect on the liquidation value of assets. For larger values of $\omega$, $k^{C} < k^{NC}$ since when he can commit the entrepreneur takes into account that specialization has a large impact on liquidation value.