Coordinating Traditional Media Advertising and Search Advertising

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Abstract

Search advertising, which is advertising shown alongside online search results presented by search engines such as Google, has been a rapidly growing form of online advertising in recent years. Unlike traditional media advertising, search advertising is sold to advertisers through an auction of keyword phrases. The new world of search advertising raises questions about how media planners should incorporate this advertising vehicle in their decision-making. In this paper, we analyze this critical issue using a model of competing, vertically differentiated firms. In choosing traditional advertising media, firms typically use measures such as cost-per-thousand (CPM) reached to compare different media. We find that, even when traditional advertising and search advertising have similar benefits, a firm would optimally bid more for search advertising than the maximum CPM it would pay in a world with traditional media only. Further, we find interestingly that the CPM a firm is willing to pay for traditional advertising when search advertising is an option may be higher than that when search advertising is unavailable.

Keywords: Search Advertising, Media Advertising, Online Advertising, Media Planning, Game Theory.

1. INTRODUCTION

In recent years, online advertising has become the fastest growing advertising medium in terms of ad revenues (Interactive Advertising Bureau 2013). Moreover, the growth in online advertising revenue surpasses revenue growth in most traditional media such as television, newspapers and radio, not all of which are growing. A major and growing component of online advertising is search advertising, which has been fueled by the growth of search engines like Google and Yahoo (Interactive Advertising Bureau 2013). Search advertising refers to ads that a search engine, say Google, displays alongside "organic" search results that it generates at its website. These search results relate to a keyword phrase supplied by the search engine user. The pricing mechanism for search advertising differs from that of traditional media advertising in a couple of ways. First, while the price of an ad in traditional media is typically based on the size and demographics of the audience reached by the media, an advertiser pays for a search ad only when a search engine user clicks on the ad at a website. Second, the price of traditional advertising is usually set by media firms or agreed to after negotiation with advertisers (Dukes and Gal-Or 2003) while the price of a search advertising is determined through a generalized second price (GSP) auction (Edelman et al. 2007). In a GSP auction, advertisers place bids by keyword for the right to have their ads placed alongside search results pertaining to the keyword. Advertisers, who are ranked higher based on their bids and ad quality (effectiveness), get the more attractive ad slots on the webpage. Further, the payment per click from each winning advertiser, who gets his ad displayed, is decided by the bid and ad quality of his next highest ranked bidder.

The rapid emergence of search advertising poses a challenge to advertising media planners, who seek to choose the most cost effective mix of media types (such as television or

online search ad) and media vehicles (such as a specific television show or a specific website) to achieve the desired coverage of their target audience. A critical issue for a media planner is how to choose between traditional advertising and search advertising when formulating the media plan. A popular measure which media planners use to compare different traditional media vehicles is CPM, the cost per thousand persons reached (Kotler and Keller 2009). The media planner would then favor the media vehicles with the lowest CPM for reaching the target audience, *ceteris paribus*. It is also customary to adjust the CPM for differences in "impact" of the media vehicles for a brand's advertising, where the impact measure captures qualitative and other factors pertaining to the media vehicle (Kotler and Keller 2009). For example, a billboard for a hotel on a highway may have a different impact on a traveler than a radio ad because the billboard can show an image of the hotel.

If the above procedure for comparing media vehicles could be extended to include search advertising, a media planner might compare the CPM of traditional media with the cost-per-click (CPC) in search advertising, suitably adjusted for the difference in media, to decide whether or not to undertake search advertising.[1] However, given that search advertising differs from traditional media advertising in the pricing mechanism, is the CPM in traditional media a good criterion to decide whether or not to undertake search advertising? For example, if the impact-adjusted CPM in traditional media is $50, should a firm refrain from search advertising that costs higher and reaches a similar target audience? Further, how does the availability of search advertising affect the CPM that a firm might be willing to pay for advertising in traditional media?

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[1] For example, with search advertising, consumers may be able to purchase with an additional click whereas with traditional media, consumers may need to get online or go to a store to complete the purchase. These differences between the two media can lead to different "impact" of the two media.

In this paper, we address these critical research questions by analyzing a model in which competing differentiated advertisers may advertise in traditional media and in search media (by participating in a search advertising auction) and compete in a product market. Specifically, we use a game theoretic model with vertically differentiated firms in a duopolistic market. We assume that both traditional advertising and search advertising inform consumers about the existence and quality of a firm's product consistent with past literature (Grossman and Shapiro 1984; Iyer et al. 2005). Our main finding is that rather than being simply a substitute for search advertising, traditional advertising spending can have a synergistic strategic effect on the competing firms' bids in the search advertising auction. First, the firm that spends more on traditional advertising finds it advantageous to bid more aggressively and win the search advertising auction, thereby limiting the search advertising exposure of its competitor. Second, higher traditional advertising spending by a firm may reduce its competitor's bid for an ad slot in the search ad auction, thus reducing search advertising costs for the first firm. A surprising result is that the second effect makes the traditional advertising leader willing to pay an even higher CPM for traditional media, even though search advertising offers a substitute vehicle.

Our analysis also shows that a firm's winning bid in the search auction would typically be higher than the maximum CPM that the firm would pay for traditional media in a world without search advertising. However, the cost-per-click (CPC) that the winning firm pays in equilibrium for a search ad would be independent of the CPM it pays concurrently for a traditional ad. The rationale for this result is that the CPM for traditional media is set by the broader competition among advertisers across many categories while the CPC for the keyword-driven search ad is determined by competition between advertisers in the focal category alone as

only these advertisers find the keyword relevant to their business. Finally, our analysis shows that the comparison of firms' profit in worlds with and without search advertising is ambiguous.

Our paper is part of an emerging academic literature on search advertising. This literature includes the seminal studies of Edelman et al. (2007) and Varian (2007), who analyze the equilibrium of the GSP auction mechanism used in search advertising. Feng et al. (2005) and Balachander et al. (2009) examine alternative GSP auction policies for determining bidders' ranks and payments per click. Edelman and Schwarz (2007) study the minimum bid requirements set by search engines in GSP auctions, while Kim et al. (2012) study the issue of the optimal number of ad slots to auction off for a keyword. Athey and Ellison (2011), Chen and He (2011), and Jerath et al. (2011) study how consumer search patterns relate to a search advertising auction. Desai et al. (2014) study advertisers' strategy of buying branded keywords in the search advertising auction while Katona and Sarvary (2010) examine how the organic search results and sponsored advertising results would be determined together in equilibrium. While these studies focus purely on search advertising, we consider the coordination between traditional advertising and search advertising by competing firms while also considering the effect of such advertising on price competition in the product market. Joo et al. (2014) and Sayedi et al. (2014) also consider the use of traditional advertising and search advertising by firms. Studying the financial services industry, Joo et al. (2013) empirically show that television advertising increases the number of related Google searches, including searches for the advertising brand. Sayedi et al. (2014) analytically study the effect of traditional advertising on the keyword bidding strategies in search advertising. Specifically, in their model, a firm's traditional advertising creates awareness among potential buyers about relevant keywords such as the product's name, and competitors may poach these potential buyers through search

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advertising by bidding on these keywords. While Sayedi et al. (2014) thus focus on the effect of traditional advertising on the bidding mechanics in search advertising, we focus on broader issues of how the acceptable cost per customer reached would compare across traditional and search media.

Other studies in the literature have analyzed competition on traditional advertising spending between firms. Grossman and Shapiro (1984) find that equilibrium industry advertising spending in a differentiated product market is excessive and further show a negative relationship between advertising spending and price. Soberman (2004) extends Grossman and Shapiro (1984) to find that an increase in advertising can increase prices as well, depending on the level of differentiation between competing firms. Iyer et al. (2005) find that when competing firms have the ability to target advertising, they would advertise more to consumers who prefer their respective products as a way of reducing price competition in the market.[2] While these studies assume a purely informative role for advertising, Chen et al. (2009) find that advertising that shifts consumer preferences can also increase price competition between firms. In contrast to these studies, we consider competition between firms on both traditional advertising and search advertising.[3] There have also been a number of models developed in marketing to help managers choose media to advertise (see Little and Lodish 1969; Aaker 1975; Zufryden 1975). The insights from our paper may be helpful in developing media planning models that incorporate search advertising as well as traditional advertising.

The rest of the paper is organized as follows. After describing the analytical model in Section 2, we analyze in Section 3 a benchmark case in which firms compete only on traditional

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[2] Villas-Boas (1993) studies the timing of advertising by competing firms and finds that firms would prefer to alternately "pulse" their advertising.

[3] There is also a stream of research which considers media providers as strategic competitors unlike our study (Dukes and Gal-Or 2003; Gal-Or and Dukes 2003; Godes et al. 2009).

advertising. Section 4 then analyzes the full model, with firms competing on both traditional and search advertising. Finally, Section 5 concludes the paper. All proofs are provided in a technical appendix.

2. MODEL

2.1 Consumers, Firms and Advertising

We consider a differentiated product market consisting of two firms, a high-quality firm H with a quality level m_H, and a low-quality firm L with a quality level m_L. We denote the quality difference $m_H - m_L$ by d. To focus on demand-side issues, we assume that the marginal cost of production is zero for both firms.

There is a unit mass of consumers, who purchase at most one unit of the product, and whose valuations for a unit of product quality are uniformly distributed between θ and $\theta + k$. For a consumer, whose valuation for a unit of quality is x, the surplus from buying product i at price p_i is $m_i x - p_i$, $i = L, H$. Consistent with the past literature (Butters 1977, Grossman and Shapiro 1984, Iyer et al. 2005), we assume that the role of advertising is to inform consumers about a product's existence, its quality (and possibly its price) and that consumers consider purchasing a product only if they are aware of it.[4] From amongst the products that a consumer is aware of, he or she purchases the firm's product that offers the highest surplus as long as this surplus is non-negative. If a consumer does not become aware of any product through advertising, she does not purchase any product.

Consumers can become aware of a product through advertising from two media: traditional advertising media (such as TV and magazines) and search advertising media (such as Google and Yahoo). In this paper, we assume without loss of generality that a consumer always

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[4] This assumption about how advertising works may be consistent with an informative role for advertising (Tirole, 1988). Another view is that advertising has a persuasive effect and does so by changing consumers' tastes such as their willingness to pay for quality (Chen et al. 2009)

uses a search engine (and is exposed to search advertising) to see if there exist any other alternatives, which could give them higher surplus. We now discuss further details of our assumptions with respect to each of these media.

Consistent with past literature, we assume that a firm's advertising expenditure, δ_i, $i = L$, H, in traditional media increases awareness of its product among consumers. Adopting a stylized response function, we assume that a firm i can choose to spend one of two amounts on traditional media, $\{0, C\}$, and achieve corresponding awareness levels of its product given by $\{\phi, \gamma\}$, where $0 < \phi < \gamma$. Villas-Boas (1993) and Iyer et al. (2005) assume a similar advertising response function and argue that such a function would be consistent with an S-shaped advertising response function that has been suggested by several papers (see for e.g. Sasieni 1971, 1989; Lodish 1971). We assume that the magnitude of C (i.e. how expensive it is to advertise on traditional media to reach γ consumers) is determined by a perfectly competitive market for traditional advertising media. Note that we assume that firms have a base level of consumer awareness ϕ, which is non-zero. [5] In other words, some consumers may already be aware of the products before being exposed to firms' ads because of past advertising or because of other activities such as visiting a store (Dukes and Gal-Or, 2003). We also assume that ϕ and γ are identical across firms, implying that firms are symmetric in the effectiveness of traditional advertising. This assumption helps us to abstract away from effects of firm asymmetries in advertising effectiveness (cf. Grossman and Shapiro 1984). For simplicity, we assume that a consumer's awareness of either product is independent of each other and that exposure to a firm's advertising in traditional media is independent of the consumer's awareness of either product.

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[5] It is a necessary assumption for the second order condition of profit functions to be satisfied.

In our model, it is useful to consider the cost of making an additional consumer aware of a firm's product through traditional advertising. Given our assumptions, this cost is given by $C/(\gamma - \phi)$ and is akin to the CPM used by media planners. We let $\tau = C/(\gamma - \phi)$ and will refer to τ as the CPM in the rest of the paper.

With respect to search advertising, we assume that there exists a search advertising provider, such as a search engine, who invites bids from advertisers for the right to advertise in one ad slot to be placed alongside search results related to the product category. Firms then decide whether or not to participate and compete in the search advertising auction. Define σ_i to be the search advertising strategy of the firm i. Then, $\sigma_i = S$ if firm i undertakes search advertising and $\sigma_i = NS$ if firm i does not undertake search advertising. Because our product market consists of only two firms, we assume that these two firms are the only interested advertisers for this slot. Our assumption of a single advertising slot is consistent with past work on GSP auctions that assume the number of slots to be one less than the number of advertisers (e.g. Edelman et al. 2007, Varian 2007). Note that with only one ad slot, the GSP auction reduces to a standard second-price auction. Consistent with the GSP auction practices for search advertising, we make the following assumptions – these are also standard assumptions in the search engine literature. First, a firm pays a price s, which we call CPC in this paper, for each consumer click of its ad when it is featured in the slot. Second, we assume that the search engine observes each advertiser's bid per click (b_i, $b_i \geq 0$) and ad quality as measured by proportion of consumers who click the advertiser's ad (click-through rate). The search engine ranks advertisers based on its expected profit per click, as measured by the product of b_i and click-through rate. The search engine then features the higher-ranked advertiser (higher expected profit) in the ad slot, provided the advertiser's bid, b_i exceeds a minimum amount, $\underline{s}$, specified by the search

engine.[6] If both advertisers bid more than $\underline{s}$, the winning firm pays an amount per click that would yield the same profit as the second-place bidder. If only one firm participates and bids more than $\underline{s}$, this firm wins the ad slot and pays $\underline{s}$ as the cost per click. For mathematical simplicity and without loss of generality, we assume that the minimum bid is arbitrarily small $(\underline{s} \to 0)$.

We now discuss the implications of our assumptions for consumer awareness. Denote the fraction of consumers in the population who are aware of product i by a_i, $i = L, H$. Prior to performing an online search, $a_H a_L$ of consumers are fully aware of both product H and L, $a_H(1 - a_L)$ and $a_L(1 - a_H)$ of them are partially aware of either H or L, and $(1 - a_H)(1 - a_L)$ of them are aware of neither H nor L. These observations follow from our assumption about the independence of the probability of awareness of each product for a consumer prior to performing an online search. When performing an online search, consumers encounter the search ad if the search engine features an advertiser in the ad slot. If search advertising features the firm that the consumer is not aware of, the consumer clicks on the ad to learn about the firm's product, and the firm is correspondingly charged for the click by the search engine. In this case, this consumer becomes aware of both products if she is aware of the other product before searching. If this consumer is not aware of any product prior to searching, she now becomes aware of the product she clicks on. However, if she is already aware of the firm featured in the search ad slot, she does not gain any new information from search advertising, and thus does not click on it.

We would like to point out that our assumptions about search advertising ignores the possibility that consumers who use a search engine may learn about the existence of products without the benefit of search advertising, because of the organic search results provided by a

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[6] An earlier version of this paper analyzes the old Yahoo ad auction, in which the search engine only compares the bids when ranking advertisers. We found our main results were similar.

search engine. We believe that awareness created in this fashion is unlikely to change the main results as long as there exist some consumers who learn mainly through search advertising. Indeed, Yao and Mela (2010), find that a significant number of consumers regularly click on the search ad slots. Moreover, awareness created through organic search results of a search engine can be considered as part of the base awareness, ϕ, in our model.

2.2 Game

The game has three stages. At the first stage, firms decide how much to spend on traditional advertising. The subgames that originate at the first stage are depicted in Table 1. At the second stage, firms decide if they will participate in search advertising, and if they do, the amount, b_i, to bid for the search ad slot. Thus, each first-stage subgame in Table 1 has a game of search advertising competition at the second stage, as described in Table 2, with *S* (*NS*) denoting that a firm bids for (does not bid for) search advertising. At the third stage, firms set prices for their products and consumers make their purchase decisions.

We assume that firms make search advertising decisions after traditional advertising decisions because the latter decisions usually need to be made in advance, while advertisers can change search advertising decisions instantaneously by changing their bids. We solve for the subgame perfect equilibrium of the game while solving for the trembling-hand perfect equilibrium in search advertising participation decisions for the second stage. The trembling-hand perfect equilibrium is a refinement of Nash equilibrium that rules out intuitively unappealing equilibria. Contingent on firms' participation in the search advertising auction, we obtain equilibrium bids in the auction using the "dominant" envy-free equilibrium (cf. Balachander et al. 2009, Edelman et al. 2007). With one ad slot as we have in our model, the

"dominant" envy-free equilibrium reduces to the dominant-strategy equilibrium of a standard second-price auction.

Finally, we make a couple of technical assumptions, which are fairly standard in the literature. First, we assume that the lowest consumer valuation for quality, θ, is sufficiently low ($\theta < k + \frac{1}{\gamma} - 1$) that the low-quality firm makes positive sales in equilibrium. Second, we assume that both firms compete for consumers who are aware of both products in equilibrium rather than sell only to consumers who are exclusively aware of the firm's product. This assumption implies an upper bound $\overline{m}_L$ and $\overline{m}_H$ for m_L and m_H, where

$$\overline{m}_L = \min\{\frac{4d(\gamma(2+\theta-k)-2)^2}{9(k+\theta)^2\ (1-\gamma)\gamma}, \frac{4d(\phi(2+\theta-k)-2)^2}{9(k+\theta)^2\ (1-\phi)\phi}\}\ ,\ \ \overline{m}_H = \min\{\frac{4d(2-\gamma(2-\theta-2k))^2}{9(k+\theta)^2\ (1-\gamma)\gamma}, \frac{4d(2-\phi(2-\theta-k))^2}{9(k+\theta)^2\ (1-\phi)\phi}\}\ .\ ^7$$

Third, we assume that valuations of consumers are sufficiently high that the market is completely covered in equilibrium, meaning that every consumer who is aware of some product makes a purchase. This assumption implies a lower bound $\underline{m}_L$ for m_L, where $\underline{m}_L = \frac{d(3-(3+\theta-k)\phi)}{3\theta\phi}$. Thus, for the rest of our analysis, we assume $\underline{m}_L < m_L < \overline{m}_L$ and $m_H < \overline{m}_H$.

3. BENCHMARK ANALYSIS: WHEN FIRMS COMPETE ONLY ON TRADITIONAL ADVERTISING

To better understand the role of search advertising in firms' media decisions, we analyze first a benchmark model where firms compete only in traditional advertising. The game in this case is a two-stage game without search advertising. Thus, firms decide how much to spend on traditional advertising in the first stage and set prices in the second stage. Table 3 summarizes

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[7] We compute this upper bound for m_L and m_H as follows. For every subgame of the full model, we obtain an upper bound of m_L and m_H that ensures that the firms compete for consumers who are aware of both products. $\overline{m}_L$ and $\overline{m}_H$ are the lowest upper bounds across all subgames. We follow a similar procedure for computing $\underline{m}_L$.

the equilibrium results and Proposition 1 presents the equilibrium strategies of firms H and L as a function of the CPM of traditional advertising.

Proposition 1: *When firms use traditional advertising alone, the subgame perfect equilibrium strategy pairs (δ_L, δ_H) are: (i) (C,C) when $\tau < \tau < \tau_1$; (ii) (0, C) when $\tau_1 < \tau < \tau_4$; (iii) (C,0) when $\tau_2 < \tau < \tau_3$; and (iv) (0,0) when $\tau > \tau_4$. Further, $\tau_1 < \tau_2$ and $\tau_3 < \tau_4$, where*

$$\tau_1 = \frac{d}{k} \left[\frac{(-2+\gamma(3+\theta-k))^2}{9\gamma} - \frac{1}{9\phi} \right], \ \tau_2 = \frac{d}{k} \left[\frac{(2+(-3+\theta+2k)\gamma)^2}{9\gamma} - \frac{1}{9\phi} \right], \ \tau_3 = \frac{d}{k} \left[\frac{(-2+(3+\theta-k)\phi)^2}{9\phi} - \frac{1}{9\gamma} \right], \ \textit{and}$$

$$\tau_4 = \frac{d}{k} \left[\frac{(2+(-3+\theta+2k)\phi)^2}{9\phi} - \frac{1}{9\gamma} \right]. \ \textit{The (C,C) equilibrium is a prisoner's dilemma.}$$

[Insert Table 3]

Recall that τ $(=C/(\gamma - \phi))$ is the traditional advertising costs for creating incremental awareness in one consumer (the CPM). Proposition 1 shows that when the CPM is low, both firms advertise in equilibrium, while neither firm advertises when the CPM is very high. For intermediate values for τ, the equilibrium would have only one of the firms advertising although the equilibrium, $(0,C)$, with only firm H advertising prevails for a wider range of τ values. For $\tau \in (\tau_2, \tau_3)$, $(C,0)$ or $(0,C)$ could each be an equilibrium. These results are quite intuitive in the sense that firms do not advertise at higher CPM values, although the high-quality firm has a higher threshold value (τ_4) of CPM above which it does not advertise in comparison to the low-quality firm (threshold value of τ_3).

In arriving at their equilibrium advertising strategies, firms consider the additional awareness created and consequent increase in demand, which we call the *demand effect*. However, increased advertising leads to an increase in the intensity of price competition resulting in the *price competition effect*. Specifically, when a firm increases consumer awareness through advertising, the proportion of consumers who are aware of both firms' products increases. We

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term the proportion of consumers who are aware of both products as the *joint awareness*. Because consumers in the joint awareness set compare prices of both products before purchase, the firm's (and competitor's) demand becomes more elastic to price changes when the proportion of such consumers increases. Consequently, the firm finds it optimal to reduce price, which causes its competitor to reduce its price as well, thereby intensifying the price competition. Thus, firm's payoffs from increasing advertising depend on the tradeoff between the sales gain stemming from the demand effect and the lower profit margin from the price competition effect (cf. Grossman Shapiro 1984, Amaldoss and He 2010). If the CPM is sufficiently low ($\tau < \tau_1$), the equilibrium results in a prisoner's dilemma with both firms advertising but making lower profits than they would if neither of them advertises.

Figure 1 plots the equilibrium regions as a function of τ and γ. In this figure, τ_1 and τ_2 are the boundaries where the $(0,C)$ and $(C,0)$ equilibria respectively cease to exist because of profitable deviation to spending C on traditional advertising by one of the firms. Along these boundaries, as γ increases for a given τ, the equilibrium has only one firm advertising rather than both firms advertising. This seemingly counterintuitive result can be explained by the aforementioned price competition effect of increased awareness. When both firms advertise, the joint awareness is γ^2, while the joint awareness drops to $\gamma\phi$ if one of the firms does not advertise. Therefore, a measure of the drop in the intensity of price competition when one firm does not advertise is $\gamma^2 - \gamma\phi$, or equivalently, $\gamma(\gamma - \phi)$. Thus, as γ increases for a given ϕ, the intensity of competition declines more steeply if one of the firms refrains from advertising, with a concomitant positive effect on profits. This positive effect overcomes any negative fallout from the demand effect arising from an increase in γ. Thus, as γ increases at the τ_1 and τ_2 boundaries, the $(C,0)$ and $(0,C)$ equilibrium, respectively, becomes feasible. Likewise, because a lower ϕ

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causes the joint awareness to decline more steeply when going from (C,C) to either $(C,0)$ or $(0,C)$, a similar argument based on the price competition effect establishes that the $(C,0)$ or $(0,C)$ equilibria are favored at small values of ϕ.

In contrast to τ_1 and τ_2, τ_3 and τ_4 represent the boundaries where the $(0,C)$ and $(C,0)$ equilibria cease to exist because the firm spending C finds it profitable to reduce spending to zero. At these boundaries, the advertising firm deviates to zero spending as γ increases or ϕ decreases, in contrast to the boundaries of τ_1 and τ_2. The difference arises in this case because the demand effect overcomes a weaker price competition effect.[8]

Finally, note that $(0,C)$ is an equilibrium for a broader set of parameter values in comparison to $(C,0)$. This result obtains because firm H enjoys higher demand and a higher profit margin because of its quality. Regardless of the price competition effect, this *quality effect* enables firm H to profitably advertise for a broader set of parameters and to restrict the parameter values for which firm L could be the lone advertiser in equilibrium.

We now present some comparative statics results in the benchmark model. In all subgames, prices and profits increase with d as a higher level of differentiation relaxes price competition. A higher θ increases prices, quantities and profits of the high-quality firm while decreasing the same for the low-quality firm. Note that a higher θ implies a greater valuation of quality differences by all consumers in the market, as the minimum valuation per unit of quality is θ. On the other hand, prices decrease with γ and ϕ in all subgames because a higher value for either of them intensifies price competition. The market share of firm H increases when its awareness increases. However, higher awareness does not always increase firm L's market share

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[8] Going from a $(0,C)$ or $(C,0)$ situation to $(0,0)$, the joint awareness drops from $\gamma\phi$ to ϕ^2, which translates to a price competition decline measure of $\phi(\gamma - \phi)$. This measure is smaller than the $\gamma(\gamma - \phi)$ we had at the boundaries of τ_1 and τ_2. Thus, the demand effect dominates the price competition effect in this case and drives the deviation to $(0,0)$ at the boundaries of τ_3 and τ_4. This deviation occurs when the demand effect is lower (low γ and high ϕ).

as firm *L*'s higher awareness also induces lower prices from firm *H*. Concerning firms' profits, in the (*C,C*) equilibrium, the profits of both firms decrease with γ. In the asymmetric equilibria ((0,*C*) and (*C*,0)), an increase in γ decreases the profit of the non-advertising firm but its effect on the profit of the advertising firm is ambiguous because of the competing demand and price competition effects.

4. ANALYSIS OF THE FULL MODEL

We present below the results of the search advertising subgame for each first-stage subgame before presenting the results of the full game.

4.1 Equilibrium of the Search Advertising Game

Proposition 2 summarizes the search advertising equilibrium in each of the first-stage subgames. We solve first for the equilibrium profits and bids in the search advertising auction based on the dominant-strategy equilibrium. Based on these results, we analyze firms' participation decisions in the search advertising auction using the trembling-hand perfect equilibrium. Proposition 2 below presents the results while Table 4 in this paper and Table 4A in the technical appendix present the detailed equilibrium results. Recall our notation that $\sigma_i = S$ if firm *i* undertakes search advertising and $\sigma_i = NS$ if firm *i* does not undertake search advertising.

Proposition 2*: Assume $\underline{s} \to 0$. The unique trembling-hand perfect equilibrium in search advertising participation strategies, (σ_L, σ_H) is (S,S) in the first-stage subgames (C,C), (C,0), (0,C) and (0,0), if $\underline{\theta} < \theta < \overline{\theta}$, where $\underline{\theta} = \sqrt{\frac{1}{\phi} + \frac{3\gamma - 2 - 2\gamma k}{\gamma}}$ and $\overline{\theta} = \min[\frac{2 - 3\gamma + \gamma k}{\gamma} - \sqrt{\frac{1}{\phi}}, \tilde{\theta}]$. Firm H's ad is featured in the ad slot in all subgames except for subgame (C,0) in which firm L is featured for $\gamma > \underline{\gamma}$. $\underline{\gamma}$ and $\tilde{\theta}$ are given in the proof.*

Proposition 2 shows that if the minimum bid is sufficiently small and if the level of consumer heterogeneity (θ) is neither too low nor too high, the unique trembling-hand perfect

equilibrium is (*S,S*) where both firms participate in the search ad auction. As noted earlier, the trembling-hand perfect equilibrium is a refinement of Nash equilibrium that rules out equilibria that are not intuitively appealing. For example, an additional Nash equilibrium that exists in these subgames is (*NS, S*) when firm *H* wins the ad slot and (*S, NS*) when firm *L* wins the ad slot. The rationale is that when firm *H* (firm *L*) wins the ad slot in the search advertising game, the losing firm makes the same profit whether or not it participates in search advertising because it gains no incremental awareness in either scenario. Therefore, there are multiple Nash equilibria. However, if the winning firm in the (*NS,S*) or (*S, NS*) equilibrium could "tremble," i.e. make an error by not submitting a bid with some probability, the losing firm could do better by submitting its bid (choose *S* rather than *NS*) so as to win the search advertising auction and increase its profit in case the winning firm trembles. This intuitively appealing argument helps eliminate the (*NS,S*) or (*S, NS*) equilibrium when the minimum bid is sufficiently small and when θ is not too small or large ($\underline{\theta} < \theta < \overline{\theta}$). Under these conditions, the losing firm can profit from the trembles of the winning firm. Note that in a two-player game as in the search advertising game (Table 2), the set of trembling-hand equilibria is the set of Nash equilibria, which are not weakly dominated (Osborne and Rubinstein 1994).

Proposition 2 also says that firm *H* wins the ad slot in the search advertising equilibrium whenever it spends at least as much as firm *L* in traditional advertising. This phenomenon is somewhat counterintuitive as one might expect the firm with lower awareness from traditional advertising to outbid the other firm as it might stand to gain more from search advertising. Indeed, in comparison to the other firm, the firm with lower awareness from traditional advertising gains more awareness from search advertising, if it wins the auction. In other words, the potential demand effect from winning the search advertising auction is greater for the firm

with lower awareness. However, in deciding its bid for the search advertising auction, each firm compares the profit from winning the search advertising auction to the profit it would make if it lets the other firm win the auction. Under this comparison, the firm with lower awareness from traditional advertising finds that it would increase price competition more if it wins the auction rather than lets the other firm win. In particular, let a_1 and a_2 be the awareness levels of the two firms, 1 and 2, after traditional advertising, with $a_1 > a_2$. If firm 1 beats firm 2 in the search advertising auction, the joint awareness (proportion of consumers who are aware of both products) would be a_2.[9] In contrast, if firm 1 had lost the auction to firm 2, the corresponding proportion would be a_1. Because the intensity of price competition increases with the joint awareness, firm 1 would face greater price competition if it loses the auction in comparison to when it wins the auction. Therefore, firm 1 bids more aggressively to win the advertising auction as it gains more profit from avoiding the higher price competition that comes from losing the auction. In sum, the price competition effect favors winning of the auction by the firm with higher awareness from traditional advertising.

However, there is also a second effect that determines how the firms bid in the auction. For any intensity of price competition (as indicated by the joint awareness), firm H does better than firm L because of the former's quality advantage. This quality effect thus favors firm H winning the search ad auction because of the firm's better profitability after advertising. When firm H has equal or greater awareness after traditional advertising than firm L (subgames $(0,0)$, $(0,C)$ and (C,C)), both the price competition effect and the quality effect favor firm H (at least weakly in some cases). Thus, firm H wins the advertising auction in these subgames. However, in subgame $(C,0)$, firm L has higher awareness than firm H after traditional advertising. Thus, the

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[9] The probability that a consumer is aware of both products is a_2 multiplied by 1 (the awareness level of firm 1 after winning the search advertising auction).

price competition effect in this case favors firm *L* winning the auction while the quality effect favors firm *H*. The price competition effect is stronger when γ is high: the joint awareness is ϕ when firm *L* wins the auction while it is γ when firm *H* wins the auction. Thus, for high values of γ in subgame (*C,0*), firm *L* has more incentive to win the auction because of a stronger price competition effect and this effect overwhelms the quality effect when $\gamma > \underline{\gamma}$.

As noted in the comparative statics for the benchmark model, profit margins increase (decrease) for firm *H* (firm *L*) with θ. Therefore, firm *H* becomes less interested in search advertising at low θ, while the reverse is true for firm *L*. Thus, trembles can eliminate the other equilibria, (*NS,S*) and (*S,NS*), only when θ is moderate as in Proposition 2. We note that when $\theta > \max(\underline{\theta}, \overline{\theta})$, we would have (*NS,S*) as the unique trembling-hand perfect equilibrium in subgames (0,0), (*C,0*), (0,*C*) and (*C,C*) and when $\theta < \min(\underline{\theta}, \overline{\theta})$, (*S,NS*) would be the unique trembling-hand perfect equilibrium in subgame (*C,0*). For ease of exposition, we focus on the parameter region in Proposition 2 that gives us the interesting case of (*S,S*) being the unique trembling-hand perfect equilibrium.

In summary, Proposition 2 underlines a *synergistic strategic effect* that traditional advertising brings to search advertising auction competition. Specifically, the firm that leads in traditional advertising gains an advantage in bidding for search advertising, *ceteris paribus*, and this advantage can allow it to win the search advertising auction. By winning the search advertising auction, a firm can reach more consumers while limiting the reach of its competitor. This strategic effect of exclusion that traditional advertising brings to search advertising is because of the unique characteristic of search advertising, where keywords are the gateway for reaching prospective customers, and advertisers who bid higher for these keywords can limit the reach of competitors.

4.2 Sub-game Perfect Equilibrium of the Overall Game

We now solve for the first-stage game decisions on traditional media spending given the equilibrium results in the second stage. Proposition 3 shows the equilibrium as a function of the CPM of traditional advertising. The equilibrium results such as profits, prices, and demands, are identical to those in the corresponding subgames in Proposition 2.

Proposition 3*: When $\underline{\theta} < \theta < \overline{\theta}$ and $\underline{s} \to 0$, the subgame perfect equilibrium strategy pairs (δ_L, δ_H) for the overall game are: (C,C) when $\tau < \underline{\tau}$, (0,C) when $\underline{\tau} < \tau < \overline{\tau}$, (C,0) when $\tau^* < \tau < \tau^{**}$ and $\gamma > \underline{\gamma}$, and (0,0) when $\tau > \overline{\tau}$, where $\tau^* = \frac{d}{k}\left[\frac{(-1+\theta+2k)^2}{9} - \frac{1}{\gamma\phi} + \frac{2(1-\gamma)(2\theta+k)}{\gamma-\phi}\right], \tau^{**} = \frac{d}{k}\left[\frac{-4+6k-(\theta+2k)^2}{9} - \frac{2(1-\gamma)(2\theta+k)}{9(\gamma-\phi)} + \frac{4}{9\gamma\phi}\right]$, $\underline{\tau} = \frac{d}{k}\left[\frac{(1+\theta-k)^2}{9} - \frac{1}{9\gamma\phi}\right]$, $\overline{\tau} = \frac{d}{k}\left[\frac{4}{9\gamma\phi} - \frac{(2+\theta-k)^2}{9}\right]$, with $\underline{\tau} < \tau^*$ and $\tau^{**} < \overline{\tau}$. Firm H is featured in the search ad slot in (C,C), (0,C), (0,0), while firm L is featured in the search ad slot in (C,0).*

Proposition 3 shows that when firms can undertake search advertising, both firms use traditional advertising when the CPM is very low ($\tau < \underline{\tau}$), while firms do not use traditional advertising when the CPM is too expensive ($\tau > \overline{\tau}$). When the CPM is in an intermediate range, equilibrium involves only one of the firms using traditional advertising, with the (0,C) equilibrium possible for a broader range of CPM than the (C,0) equilibrium. These results are broadly similar to those in the benchmark case as can be seen by comparing Figure 2, which plots the equilibrium regions for the full model, with Figure 1. Consistent with our discussion earlier of the search advertising subgame, firm H wins the search ad auction in the (C,C), (0,C) and (0,0) equilibria of the full model while firm L wins the auction in the (C,0) equilibrium. The limits on θ in Proposition 3 are the technical conditions imposed in Proposition 2 to ensure a

trembling-hand perfect equilibrium in which both firms participate in the search advertising auction.

4.3 Analysis of the Equilibrium Results of the Full Model

We now examine some implications of the propositions and some comparative statics. First, it is interesting to compare the maximum CPM that a firm, which uses traditional advertising, is willing to pay under the full model as opposed to the benchmark case.

Result 1: (a) In the asymmetric $(C,0)$ and $(0,C)$ equilibria of the full model, the maximum CPM that the firm using traditional advertising is willing to pay exceeds that in the corresponding equilibrium of the benchmark case, when θ is sufficiently low. Specifically, in the $(C,0)$ equilibrium, $\tau^{**} > \tau_3$ when $\theta < \theta_1$ and in the $(0,C)$ equilibrium, $\bar{\tau} > \tau_4$ when $\theta < \theta_2$, where θ_1 and θ_2 are given in the Appendix.

(b) The maximum CPM, $\bar{\tau}$, that any firm is willing to pay for traditional advertising in the full model exceeds that (τ_4) in the benchmark case when $\theta < \theta_2$.

One might expect that the option of engaging in search advertising would reduce the CPM that firms may be willing to pay for traditional advertising because of the substitution effect. However, Result 1 suggests surprisingly that the maximum CPM that a firm would pay for traditional advertising may increase when search advertising is available. Specifically, as Result 1 (a) shows, the firm using traditional advertising in the $(C,0)$ and $(0,C)$ equilibria may be willing to pay a higher CPM than in the corresponding equilibrium of the benchmark case. Result 1 (b) follows from Result 1 (a) to suggest that a higher CPM may be paid for traditional advertising when search advertising becomes available.

To see the intuition, consider the comparison between $\bar{\tau}$ and τ_4 for the $(0,C)$ equilibrium. In order to do so, we examine the profit difference for firm H between the $(0,C)$ and $(0,0)$

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subgames in each case, as this difference determines τ_4 and $\bar{\tau}$. We denote firm H's profit in the benchmark case for the (0,0) and (0,C) subgames to be $\pi_H^{BM}(0,0)$ and $\pi_H^{BM}(0,C)$ respectively. The corresponding notation for firm H's profit in the search advertising game is $\pi_H(0,0)$ and $\pi_H(0,C)$ respectively. Further, we use the notation $\Omega_i(a_L, a_H)$ to denote firm i's gross profit before subtracting advertising costs, where $i = L$, H, and a_L and a_H are the awareness levels of firms L and H respectively. With the above notation, the difference in payoff for firm H between the (0,C) and (0,0) subgames in the benchmark case is the following:

$$\pi_H^{BM}(0,C) - \pi_H^{BM}(0,0) = \Omega_H(\phi,\gamma) - \tau(\gamma - \phi) - \Omega_H(\phi,\phi) \tag{1}$$

Note that $\tau(\gamma - \phi)$ is firm H's cost of traditional advertising in the (0,C) subgame, where τ is the CPM and $\gamma - \phi$ is the fraction of new consumers made aware by the traditional advertising. For (0,C) to be an equilibrium, equation (1) has to be positive which yields the following expression for τ_4:

$$\tau \leq \frac{\Omega_H(\phi,\gamma) - \Omega_H(\phi,\phi)}{(\gamma - \phi)} = \tau_4 \tag{2}$$

For the search advertising game, the difference in firm H's payoffs between the (0,C) and (0,0) subgames is the following:

$$\pi_H(0,C) - \pi_H(0,0) = \left[\Omega_H(\phi,1) - \tau(\gamma - \phi) - b_L^{(0,C)}(1 - \phi)\right] - \left[\Omega_H(\phi,1) - b_L^{(0,0)}(1 - \phi)\right]$$

$$= b_L^{(0,0)}(1 - \phi) - b_L^{(0,C)}(1 - \phi) - \tau(\gamma - \phi) \tag{3}$$

In the above expression, $b_L^{(0,C)}$ and $b_L^{(0,0)}$ represent respectively the bids of the losing firm L in the search advertising game. Note that the profit expressions for both subgames in the right-hand side of equation (3) account for the cost to firm H from successfully winning the search advertising auction. In either subgame, firm H pays an amount equal to the expected profit to the search engine if it were to accept the bid of the losing firm, L. For example, in subgame (0,C),

the search engine expects a profit of $b_L^{(0,C)}(1-\phi)$ if it awards the search ad slot to the losing

firm L, as the consumers, who click, are those $1-\phi$ consumers, who are unaware of firm L. In

the dominant strategy equilibrium of the search ad auction, firm L's equilibrium strategy is to bid

its value for the advertising exposures from the search ad. Thus, it would bid such that its cost of

the search ads would be no more than the profit difference from winning and losing the auction.

Therefore, we have the following equations concerning $b_L^{(0,C)}$ and $b_L^{(0,0)}$:

$$b_L^{(0,C)}(1-\phi) = \Omega_L(1,\gamma) - \Omega_L(\phi,1) \tag{4}$$

$$b_L^{(0,0)}(1-\phi) = \Omega_L(1,\phi) - \Omega_L(\phi,1) \tag{5}$$

Note that the only difference between the right-hand expressions in equations (4) and (5)

is the awareness levels of firm H when firm L wins the auction, which are γ and ϕ respectively

in equations (4) and (5). Substitution of equations (4) and (5) in equation (3) yields the

following:

$$\pi_H(0,C) - \pi_H(0,0) = \Omega_L(1,\phi) - \Omega_L(1,\gamma) - \tau(\gamma-\phi) \tag{6}$$

For (0,C) to be an equilibrium in the search advertising game, equation (6) should be

positive resulting in the following condition:

$$\tau \le \frac{\Omega_L(1,\phi) - \Omega_L(1,\gamma)}{(\gamma-\phi)} = \bar{\tau} \tag{7}$$

Comparison of equations (1) and (3) emphasizes the intuition behind Result 1 that the

maximum CPM that firm H is willing to pay in the benchmark game is determined by the

comparison of the profit it makes in the (0,C) and (0,0) subgames (i.e. its returns in the product

market stemming from the traditional advertising). In contrast, the maximum CPM that firm H is

willing to pay for traditional advertising in the full model is determined by the strategic *effect* of

traditional advertising spending (in addition to the exclusion effect seen earlier) on firm L's bids

in the search advertising auction. The rationale is that firm L's bids influence firm H's search advertising costs because of the second-price auction format. In particular, firm L bids higher in subgame $(0,0)$ in comparison to subgame $(0,C)$. In subgame $(0,0)$ the joint awareness is ϕ whether or not firm L wins the search advertising auction, implying that the intensity of price competition is unaffected no matter which firm wins the search auction. However, in subgame $(0,C)$, the joint awareness increases to γ from ϕ if firm L, rather than firm H, wins the auction. The concomitant increase in the intensity of price competition if firm L wins induces firm L to bid less aggressively in subgame $(0,C)$ in comparison to subgame $(0,0)$. Thus, in the full model, firm H's value for traditional advertising is driven by how much it can decrease firm L's bids in the search ad auction by spending C on traditional advertising.

To understand why $\bar{\tau}$ can be greater than τ_4 for smaller θ, compare equations (2) and (7). τ_4 is proportional to the change in firm H's profit when its awareness increases from ϕ to γ (holding firm L's awareness at ϕ), while $\bar{\tau}$ is proportional to the decrease in firm L's product-market profits when firm H's awareness increases by the same amount (while holding firm L's awareness at 1). In other words, τ_4 in the benchmark case is driven by firm H's own profit, while $\bar{\tau}$ in the full model is driven by its competitor's profit, as this quantity affects the competitor's bidding. As seen in the benchmark case, a lower θ (lower quality valuation in the market) decreases firm H's profit levels while increasing firm L's profit levels. Therefore, $\bar{\tau}$ exceeds τ_4 when θ is smaller than θ_2. Further, because θ_2 decreases with γ, $\bar{\tau}$ exceeds τ_4 when γ is lower, i.e. when traditional advertising is less effective in increasing awareness. Thus, lower effectiveness of traditional advertising (lower γ) paradoxically leads to a high-quality firm being more willing to pay for traditional advertising when search advertising becomes available.

The comparison in Result 1 relies on the assumption that the base awareness, ϕ, in the benchmark case and the full model remains the same. It is possible that the base awareness is higher in the full model because of awareness created through organic search results of a search engine. The expression for $\bar{\tau}$ suggests that such an increase in ϕ would tend to reduce $\bar{\tau}$.

While Result 1 focuses on equilibria in which one firm does not advertise, a pertinent question is whether firms become willing to pay a higher or lower CPM under the full model in equilibria in which both firms use traditional advertising. Result 2 provides the answer.

Result 2: The maximum CPM, $\underline{\tau}$, that can support a (C,C) equilibrium in the full model is smaller than the corresponding value, τ_1, in the benchmark case.

Thus Result 2 suggests that in a world with search advertising, the CPM needs to be lower for both firms to use advertising in equilibrium. The ability of search advertising to reach additional consumers increases the intensity of price competition, causing firm L to otherwise avoid traditional advertising spending in order to mitigate price competition.

Results 1 and 2 examine how the maximum CPM that a firm may be willing to pay for traditional advertising changes in a world with search advertising. A related question is whether a firm may bid more for search advertising than the maximum CPM it would be willing to pay in a world without search advertising. In other words, how do firms' bid for search advertising compare with the maximum CPM they would pay in the benchmark case? Result 3 provides the answer to this question.

Result 3: The winning bid in the search auction is typically higher than the maximum CPM that the winning firm would pay in a similar equilibrium of the benchmark case, i. e. $\tau_3 < b_L^{(C,0)} =$

$$\frac{d}{9(1-\gamma)k}\left[\left(\frac{2}{\phi^2}-\frac{2+\theta-k}{\phi}\right)^2-\left(\frac{1}{\gamma^2}-\frac{1+\theta-k}{\gamma}\right)^2\right], \tau_1 < b_H^{(C,C)}=\frac{d}{k}\left[\frac{1}{3\gamma}-\frac{3-2\theta-4k}{9}\right] \text{ and for } \theta \text{ sufficiently}$$

small, $\tau_4 < b_H^{(0,C)}=\frac{d}{9(1-\gamma)k}\left[\left(\frac{2}{\phi^2}-\frac{2-\theta-2k}{\phi}\right)^2-\left(\frac{1}{\gamma^2}-\frac{1-\theta-2k}{\gamma}\right)^2\right].$

Result 3 shows interestingly that the firm, which wins the search ad auction, tends to bid more per click in the auction than the maximum CPM it is willing to pay in a similar equilibrium in the benchmark case. Intuitively, the benefit to a firm using traditional advertising in the benchmark case comes from the increase in demand offset by an increase in price competition. However, winning the search ad auction in the full model allows a firm to both increase demand and avoid more intense competition by excluding its competitor from the ad slot. Thus, a firm may optimally bid more for the search ad auction than the maximum CPM it is willing to pay in the benchmark case. This result implies that firms cannot apply the same CPM criteria they employ in the traditional advertising world when bidding for search advertising. Note that our model assumes that search advertising only creates awareness and does not give any other advantages such as allowing consumers to buy the product. Thus, we show that even when search ads have the same functionality as traditional ads, a firm may bid higher for a search ad in order to strategically exclude its competitor from the search ad slot.

Result 3 compares the winning firm's search ad bid with the maximum CPM it is willing to pay for traditional advertising in the benchmark case. How does a firm's search advertising bid relate to the CPM it pays for traditional advertising when both forms of advertising are available? Result 4 answers this question.

Result 4: In the full model, a winning firm's bid or CPC in the search ad auction is independent of the CPM paid for traditional advertising. Thus, the bid or CPC in the search ad auction can be higher or lower than the CPM paid for traditional advertising.

Arguments based on economic efficiency might suggest that a firm using two media with identical effects on consumers should pay an identical cost per consumer reached for either media. By our model assumptions, traditional advertising and search advertising have identical consumer effects as both media make consumers aware and informed about a firm's product. However, Result 4 shows that a winning firm's bid and CPC can be higher or lower than the CPM paid for traditional advertising, contrary to an argument based on economic efficiency. The rationale for this result is consistent with the intuition explained earlier. The CPM for traditional advertising is determined by the broader competition for media space among firms both within the focal product category and without. However, the bid and CPC is determined by competition for the search ad slot between firms in the focal category as the auction is based on a keyword related to the focal category. In particular, the bids of both firms are driven both by the payoff from the increased awareness through the search ad slot as well as the payoff from denying their competitor from the search ad slot, as noted in Section 3. Thus, a winning firm's bid and CPC are unrelated to the CPM paid for traditional advertising, and can be higher or lower than the latter, contrary to an economic efficiency-based prescription.

Note that by our model assumptions, the reach of traditional advertising is limited in comparison to search advertising, which can potentially reach all consumers. Even with this discrepancy, an economic efficiency argument would suggest that a firm pays no more than the cost of search ad for traditional advertising. However, Result 4 suggests that such an argument is incorrect as the CPC can be lower than the cost of traditional advertising in the full equilibrium.

Our next result looks at the effect of model parameters on the possibility of a (0,0) equilibrium in the full model. In other words, when search advertising is possible, what conditions lead to both firms not spending on traditional advertising?

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Result 5: In the full model, a (0,0) equilibrium obtains if ϕ or γ is sufficiently large, or if d is sufficiently small, *ceteris paribus*.

The intuition for Result 5 is best understood by examining equation (7), which determines the boundary of the (0,0) equilibrium in Figure 2. Because product-market profits of firm L in equation (7) are proportional to d, the difference in quality between the firms (see Table 3), firm H's willingness to pay for traditional advertising to strategically limit firm L's bid in the search advertising auction reduces at lower d. Thus, an (0,0) equilibrium obtains for lower values of d. For the effect of γ and ϕ, the numerator and denominator of (7) are affected in offsetting ways. However, the numerator effect (the strategic effect on firm L's bid) is dominant for ϕ while the denominator effect (spreading traditional advertising costs over a larger reach, $\gamma - \phi$) dominates for γ. Thus, the (0,0) equilibrium region prevails for larger ϕ and γ. In contrast, for the benchmark case, an (0,0) equilibrium arises with smaller γ and larger ϕ, as noted in section 3. Finally, we examine whether or not firms receive higher profit with search advertising.

Result 6: In the (C,C) equilibrium of the full model, firm H's profit is higher, while firm L's profit is lower, than that in the benchmark case. In the (C,0), (0,C) and (0,0) equilibria of the full model, profit comparisons with the benchmark model are ambiguous.

In the (C,C) equilibrium, which arises only when τ is sufficiently small, search advertising allows firm H to gain additional awareness among consumers by winning the auction. This additional awareness makes firm H's profit higher and firm L's profit lower in the full model in comparison to the benchmark case.

4.4   Numerical Example

Assume that $\theta = 0.1$, $k = 0.15$ $d = 1$, $m_L = 32$, $m_H = 33$, $\phi = 0.24$, $\gamma = 0.4$, and $\tau = \$3.50$. We can verify that these parameters satisfies the technical conditions, which are the following:

$-0.259 < \theta < 0.109$, and $31.83 < m_L < 44.10$, $m_H < 54.80$. For this example, τ_1=-1.84, τ_2=-1.38, τ_3= \$3.30, and τ_4= \$3.99, so that $(0,C)$ is the equilibrium in the benchmark case. In the full model, $\underline{\tau}$=-7.05, τ^*=-5.50, τ^{**}=\$26.50, and $\bar{\tau}$ =\$28.05, so that both $(C,0)$ and $(0,C)$ can be an equilibrium in the full model. Consistent with Result 1, note that the maximum CPM $(\bar{\tau})$ that any firm is willing to pay for traditional advertising in the full model exceeds that (τ_4) in the benchmark case. In the search advertising auction of the $(0,C)$ equilibrium, firm H bids \$11.65 per click while firm L bids \$1.21. Firm H wins the search advertising auction and pays a CPC of \$1.53. Note that firm H bids higher than the maximum CPM of τ_4= \$3.99 that it would pay in the benchmark case in the $(0,C)$ equilibrium (see Result 3). Further, firm H's bid is higher than the CPM of \$3.50 for traditional advertising in this example (see Result 4). The equilibrium profits in the benchmark case are π_H^{BM} = \$4.26 and π_L^{BM} = \$2.75. In the full model, the equilibrium profits are π_H = \$6.58 and π_L = \$1.84. Thus, firm H is better off with search advertising while firm L is worse off , which is not a general result, however (see Result 6).

5. CONCLUSION

Our main finding is that spending on traditional advertising produces a two-fold synergistic strategic effect on competing advertisers' bids in a search ad auction. A first strategic effect arises because the firm with higher (lower) awareness from traditional advertising is more (less) eager to win the auction than the other firm. The rationale is that winning of the search ad auction by the firm with the higher rather than lower awareness limits the number of consumers aware of both products (the joint awareness set) and thus restricts price competition. We find that a high-quality firm tends to be better positioned to take advantage of this strategic effect because of its higher profit margins. A second strategic effect of traditional advertising is that it helps limit the bids of the losing firm in the search advertising auction. Because of this strategic

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effect, firms become willing to pay more for traditional advertising in a world with search advertising. This result is surprising given that search advertising offers a substitute vehicle for traditional advertising. In sum, the strategic effect on search advertising enables the traditional advertising leader to limit the exposure of its competitor and to limit the latter's bid in the search ad auction. Because of the strategic effect, the traditional advertising leader ends up restricting the overall exposure of its competitor in equilibrium even though its competitor has both traditional media and search media as potential advertising vehicles.

We also find that the winning bid in the search ad auction is typically higher than the maximum CPM that the winning firm would pay for traditional advertising in a world without search advertising. This result obtains even though we assume that search advertising and traditional advertising offer similar functionality to the advertiser. The rationale is that by winning the search ad auction, the winning firm also benefits from excluding its competitor from gaining exposure through search advertising. On the other hand, with traditional advertising, an increase in awareness for a firm does not necessarily imply a corresponding decrease in awareness for the competing firm. This crucial difference between traditional media and search media is critical to understanding the above differences in costs of the two media. While the winning firm is thus willing to bid a higher CPC in the search ad auction than the maximum CPM it is willing to pay for traditional advertising, its bid or CPC is independent of the CPM it pays for traditional advertising. The reason for this result is that the CPC of the search ad is determined by competition between advertisers in the focal category for the keyword while the CPM of traditional media is determined by the broader competition for ad space among advertisers spanning many product categories.

We now briefly discuss potential avenues for further research. In our model, we assume a similar informative role for both traditional advertising and search advertising. This feature of our model allows us to abstract away from any differences across the two media to address the issue of relative value of the two media to a firm. Future research may incorporate potential differences across media. For example, search advertising may be used by consumers later in the decision process. Another potential area for future research is to consider a positive interaction between search and traditional advertising that could potentially reinforce each media's effectiveness (cf. Joo et al. 2013). It may also be of interest to consider the coordination of traditional advertising and search advertising when advertising can change preferences (e.g. Chen et al. 2009) instead of being just informative. Lastly, while we consider vertically differentiated firms, it might be useful to analyze a model where firms are horizontally differentiated.

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Table 1. Nomenclature of the Subgames in Traditional Media Advertising Competition

		H	
		C	0
L	C	(C,C)	(C,0)
	0	(0,C)	(0,0)

Table 2. Nomenclature of the Subgames in Search Advertising Competition

		H	
		No Search Ad	Search Ad
L	No Search Ad	(NS,NS)	(NS,S)
	Search Ad	(S,NS)	(S,S)

Table 3. Equilibrium Prices, Quantities, and Profits in the Benchmark Model

	(C,C)	(C,0)	(0,C)	(0,0)
π_H^{BM}	$\frac{d\gamma^2}{k}\left[\frac{1}{\gamma} - \frac{3-\theta-2k}{3}\right]^2 - C$	$\frac{d\gamma\phi}{k}\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{(3-\theta-2k)}{3}\right]^2$	$\frac{d\gamma\phi}{k}\left[\frac{2\gamma+\phi}{3\gamma\phi} - \frac{(3-\theta-2k)}{3}\right]^2 - C$	$\frac{d}{k}\left[1 - \frac{\phi(3-\theta-2k)}{3}\right]^2$
π_L^{BM}	$\frac{d\gamma^2}{k}\left[\frac{1}{\gamma} - \frac{3+\theta-k}{3}\right]^2 - C$	$\frac{d\gamma\phi}{k}\left[\frac{\phi+2\gamma}{3\gamma\phi} - \frac{3+\theta-k}{3}\right]^2 - C$	$\frac{d\gamma\phi}{k}\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{(3+\theta-k)}{3}\right]^2$	$\frac{d}{k}\left[1 - \frac{\phi(3+\theta-k)}{3}\right]^2$
p_H^{BM}	$d\left[\frac{1}{\gamma} - \frac{3-\theta-2k}{3}\right]$	$d\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{(3-\theta-2k)}{3}\right]$	$d\left[\frac{2\gamma+\phi}{3\gamma\phi} - \frac{(3-\theta-2k)}{3}\right]$	$d\left[\frac{1}{\phi} - \frac{3-\theta-2k}{3}\right]$
p_L^{BM}	$d\left[\frac{1}{\gamma} - \frac{3+\theta-k}{3}\right]$	$d\left[\frac{\phi+2\gamma}{3\gamma\phi} - \frac{(3+\theta-k)}{3}\right]$	$d\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{(3+\theta-k)}{3}\right]$	$d\left[\frac{1}{\phi} - \frac{3+\theta-k}{3}\right]$
q_H^{BM}	$\frac{\gamma^2}{k}\left[\frac{1}{\gamma} - \frac{3-\theta-2k}{3}\right]$	$\frac{\gamma\phi}{k}\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{3-\theta-2k}{3}\right]$	$\frac{\gamma\phi}{k}\left[\frac{2\gamma+\phi}{3\gamma\phi} - \frac{(3-\theta-2k)}{3}\right]$	$\frac{\phi}{k}\left[1 - \frac{\phi(3-\theta-2k)}{3}\right]$
q_L^{BM}	$\frac{\gamma^2}{k}\left[\frac{1}{\gamma} - \frac{3+\theta-k}{3}\right]$	$\frac{\gamma\phi}{k}\left[\frac{\phi+2\gamma}{3\gamma\phi} - \frac{3+\theta-k}{3}\right]$	$\frac{\gamma\phi}{k}\left[\frac{\gamma+2\phi}{3\gamma\phi} - \frac{(3+\theta-k)}{3}\right]$	$\frac{\phi}{k}\left[1 - \frac{\phi(3+\theta-k)}{3}\right]$

Table 4. Equilibrium Prices, Quantities, and Profits in the Full Model for Select Subgames[10]

		(C,C)	(0,0)
π_H	(S,S)	$\frac{d\gamma}{k}\left[\frac{1}{9\gamma^2} + \frac{6k+\theta(6-(6-\theta)\gamma)-2}{9\gamma} + \frac{1-2k(3-2\theta-2k)}{9}\right] - C$	$\frac{d\phi}{k}\left[\frac{1}{9\phi^2} + \frac{6k+\theta(6-(6-\theta)\phi)-2}{9\phi} + \frac{1-2k(3-2\theta-2k)}{9}\right]$
	(NS,S)	$\frac{d\gamma}{k}\left[\frac{2}{3\gamma} - \frac{(2-\theta-2k)}{3}\right]^2 - C - (1-\gamma)$	$\frac{d\phi}{k}\left[\frac{2}{3\phi} + \frac{2-\theta-2k}{3}\right]^2 - \underline{s}(1-\phi)$
π_L	(S,S) / (NS,S)	$\frac{d\gamma}{k}\left[\frac{1}{3\gamma} - \frac{(1+\theta-k)}{3}\right]^2 - C$	$d\phi\left[\frac{1}{3\phi} - \frac{1+\theta-k}{3}\right]^2$
p_H		$d\left[\frac{2}{3\gamma} - \frac{(2-\theta-2k)}{3}\right]$	$d\left[\frac{2}{3\phi} + \frac{2-\theta-2k}{3}\right]$
p_L		$d\left[\frac{1}{3\gamma} - \frac{1+\theta-k}{3}\right]$	$d\left[\frac{1}{3\phi} - \frac{1+\theta-k}{3}\right]$
q_H		$\frac{\gamma}{k}\left[\frac{2}{3\gamma} - \frac{(2-\theta-2k)}{3}\right]$	$\frac{\phi}{k}\left[\frac{2}{3\phi} + \frac{2-\theta-2k}{3}\right]$
q_L		$\frac{\gamma}{k}\left[\frac{1}{3\gamma} - \frac{(1+\theta-k)}{3}\right]$	$\frac{\phi}{k}\left[\frac{1}{3\phi} - \frac{1+\theta-k}{3}\right]$
b_H	(S,S)	$\frac{d}{k}\left[\frac{1}{3\gamma} - \frac{3-2\theta-4k}{9}\right]$	$\frac{d}{k}\left[\frac{1}{3\phi} + \frac{3-2\theta-4k}{9}\right]$
	(NS,S)	$\frac{d}{k}\left[\frac{4}{9\gamma} - \frac{13-4\theta-8k}{9} + \frac{(\theta+2k-3)^2\gamma}{9}\right]$	$\frac{d}{k}\left[\frac{4}{9\phi} - \frac{13-4\theta-8k}{9} + \frac{(\theta+2k-3)^2\phi}{9}\right]$
b_L	(S,S)	$\frac{d}{k}\left[\frac{1}{3\gamma} - \frac{3+2\theta-2k}{9}\right]$	$\frac{d}{k}\left[\frac{1}{3\phi} - \frac{3+2\theta-2k}{9}\right]$

Note: The equilibrium results in the (*NS,NS*) subgames are the same as in the benchmark model.

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[10] The results for the (C,0) and (0,C) subgames are reported at the end of the technical appendix.

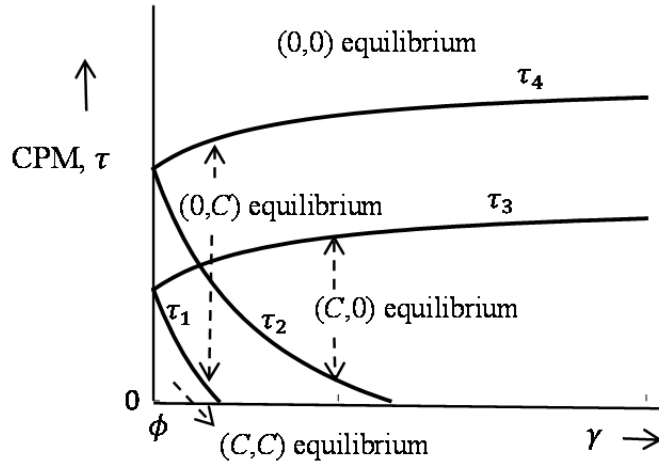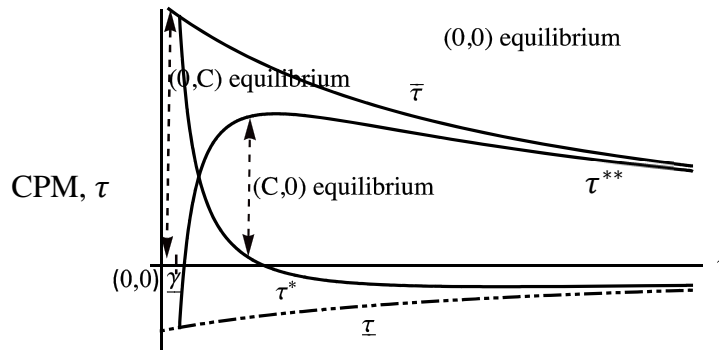**Figure 1. Boundaries of Equilibrium Regions in the Benchmark Case**

Figure 2. Boundaries of Equilibrium Regions in Full Model[11]

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[11] Note that the (C,C) equilibrium does not exist for the parameters used in Figure 2 because $\underline{\tau}$ is negative.