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An information-based theory of international currency ☆

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ABSTRACT

This paper develops an information-based theory of international currency based on search frictions, private trading histories, and imperfect recognizability of assets. Using an open-economy search model with multiple competing currencies, the value of each currency is determined without requiring agents to use a particular currency to purchase a country's goods. Strategic complementarities in portfolio choices and information acquisition decisions generate multiple equilibria with different types of payment arrangements. While some inflation can benefit the country issuing an international currency, the threat of losing international status puts an inflation discipline on the issuing country. When monetary authorities interact in a simple policy game, the temptation to inflate can lead optimal policy to deviate from the Friedman rule. The calibrated model can produce a welfare cost of losing international status for the issuing country larger than previous findings, though estimates depend critically on inflation rates and information costs.

1. Introduction

The U.S. dollar plays a central role in the international monetary system. The 2006 Treasury reports that nearly 60% of dollar banknotes are held abroad. Over the last half century, the dollar has also been the main currency used for trade invoicing, denominating international debt, and foreign exchange trades.[1] Although this current arrangement is the joint outcome of choices made by private citizens and regulations by official bodies, much of the international macroeconomic literature treats payment arrangements as given by restricting agents to only using a particular currency. While this assumption prevents the exchange rate from being indeterminate, as in Kareken and Wallace (1981), such an approach is especially unsatisfactory for understanding the conditions under which a currency can achieve international status, or how it might lose that status. Moreover, what are the implications of endogenous acceptance decisions for the choice of inflation in an open economy? And what are the welfare gains to having an international currency?

The objective of this paper is to provide a general equilibrium framework to be able to answer these questions. For that purpose, I develop a simple open-economy search model with multiple currencies to analyze three central issues in international monetary economics: (i) the conditions that allow for the emergence of an international medium of exchange, (ii) the choice of inflation by monetary authorities when currency acceptability is endogenous, and (iii) the welfare benefits of having an international currency for both the issuing country and the rest of the world. Search-theoretic models are particularly useful at addressing international currency use since they explicitly formalize the essential role of money, rather than assuming it exogenously.[2] In this spirit, this paper differs from much of the international macroeconomic literature

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[1] In addition to an international currency's role as a store of value and medium of exchange in cross-border goods transactions, which is the main focus of this paper, it can also serve to invoice imports and exports, anchor the exchange rate, effectuate cross-border payments, denominate international assets and liabilities, and facilitate interbank transactions. Many of these uses are reinforcing, and the currencies used for pricing also tend to serve as means of payment. Goldberg (2011) finds that the U.S. dollar maintains a dominant role in all key functions.

[2] Earlier search models of international currency include contributions by Matsuyama et al. (1993), Wright and Trejos (2001), Head and Shi (2003), Li and Matsui (2009), and Liu and Shi (2010), among others. As further discussed in Section 1.1, making progress on the substantive issues highlighted above requires extensions of these previous studies as none can generate the full class of results in this paper.

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by letting private citizens choose which currencies to accept as means of payment, and not fixing its role by assumption. By integrating recent advances in monetary search theory with international monetary economics, this paper attempts to further our understanding of currency competition by providing microfoundations for the internationalization of currencies.

The model features two key ingredients that capture the fact that international monetary arrangements are the dual outcomes of choices made by private citizens and regulations by official bodies. First, payment patterns are pinned down by letting private citizens choose which currencies to accept. The basic idea is quite general, dating back to Jevons (1875) and Menger (1892), and has to do with the fact that an object that is widely recognizable, such as dollars in the U.S., is better at facilitating trade than alternatives such as foreign currency.[3] For example, the fact that sellers in the U.S. are not as familiar with pesos—they might be worried that they are counterfeits—makes them reluctant to accept it as payment unless some costly information is acquired, as in Kim (1996) and Lester et al. (2012).[4] Second, *government transaction policies* are introduced in order to account for the fact that payment outcomes also reflect choices made by official bodies. Historically, a currency will not become international unless there is a centralized institution that favors its use. This is often achieved in practice by announcing legal tender status or only accepting domestic money for tax payments.[5] The basic idea is that by simply accepting a particular currency in its own trades, governments may induce private agents to do the same.

In the two-country, two-currency model, agents interact first in decentralized international trade markets and then in a Walrasian currency exchange market. The frictions in this environment are search frictions, private trading histories, and imperfect recognizability of assets. Each country issues one currency and is defined by two features: citizens in each country receive transfers of domestic currency and meet each other more frequently than they meet foreigners. Trade entails exchanging local goods for a portfolio of currencies, with no restrictions on which monies can be used between private citizens. Since what sellers accept depend on what buyers hold, and vice versa, complementarities in the trading environment lead to multiple equilibria where zero, one, or two international monies can emerge. For instance, when information costs are sufficiently high, an equilibrium with two national currencies arises endogenously, a result that is difficult to achieve in previous dual-currency search models.[6] Network externalities can lead to coordination failures, with no guarantee that the world will end up with a socially efficient monetary system.

By formalizing the role of currency in payments, the model provides a channel through which monetary policy can affect prices, trade, and welfare. For instance, currency substitution occurs as an endogenous response to local inflation: as it becomes more costly to hold local money, agents start substituting with foreign currency such as dollars. This captures the phenomenon of dollarization common in many Latin American and Eastern European economies. The theory also emphasizes an important influence on the choice of money as an international medium of exchange. Fundamentals, as well as expectations regarding other agents' behavior, jointly determine this decision and thereby determine the circulation patterns that arise. Due to inertia, it is difficult to dislodge an incumbent currency from its international role, whose use is associated with low information costs. At the same time, a temporary disruption—such as a change in inflation—can permanently shift payment patterns. International currency use therefore reflects both fundamentals and history, consistent with what we observe in practice.

This paper also explicitly models the strategic interaction among monetary authorities to obtain insights on the choice of inflation in interdependent economies. The dynamic policy game captures the tradeoffs faced by policymakers and generates an inflation Laffer curve. While some inflation can benefit the issuing country through increased seigniorage from foreigners, too much inflation lowers the purchasing power of money and hence trade between countries. At the same time, the threat of losing international status puts an inflation discipline on the issuing country, a finding that first appeared in Li and Matsui's (2009) model with indivisible monies. When monetary authorities interact in a simple policy game, the issuing country must therefore trade off the temptation to inflate with the threat of losing international status to set an inflation rate that will generally deviate from the Friedman rule, a result that is new and difficult to obtain in previous studies.

To illustrate these theoretical findings and quantify the welfare effects across countries, the model is calibrated to match international trade data. According to the theory, a country's welfare consists of seigniorage transfers across countries and the surplus due to liquidity provision to citizens net of any information costs incurred. Depending on inflation rates and the calibration strategy for the model's information costs, the welfare benefit to the U.S. of having the dollar as the sole international currency ranges from 0.4% to 1% of GDP per year, of which approximately 0.1–0.2% of GDP is due to seigniorage. These estimates thus serve as upper and lower bounds, respectively, on the welfare gains from international currency use. This can also be compared with 0.4% of GDP from Portes and Rey (2002), which only include seigniorage gains and the savings due to reduced transaction costs. This suggests that alternative studies may be underestimating the benefit of international liquidity provision since previous models do not account for the general equilibrium effects an international currency has at expanding trade opportunities abroad.

This paper proceeds as follows. Section 1.1 reviews the related literature. Section 2 describes the environment, and Section 3 defines equilibrium. Currency regimes are characterized in Section 4, which also discusses how monetary policies affect prices, allocations, and welfare. Section 5 considers a simple policy game to determine the choice inflation by monetary authorities in an open economy. Section 6 calibrates the generalized model using international trade data, and Section 7 calculates the welfare benefits of an international currency. Finally Section 8 concludes.

1.1. Related literature

Theories of international currency date back a long way, and this paper provides microfoundations for insights first articulated by Menger (1892), Kindleberger (1967), Swoboda (1969), and Krugman (1984). More recently, Rey (2001), Lyons and Moore (2009), and Devereux and Shi (2013) provide theories of vehicle currency use in foreign exchange markets, while Bacchetta and Van Wincoop (2005) and Goldberg and Tille (2008) develop theories of invoice currencies. In contrast, this paper emphasizes the role of international currencies in facilitating cross-border goods transactions and derives this role using search theory.

[3] A vast literature that explores the link between recognizability, information, and liquidity includes Brunner and Meltzer (1971), Alchian (1977), Williamson and Wright (1994), Banerjee and Maskin (1996), Berentsen and Rocheteau (2008), Lester et al. (2012), and Li et al. (2012).

[4] For instance, the rise of the U.S. dollar prior to World War I was partially due to the increasingly dubious nature of the quality of foreign bonds (see e.g. Eichengreen (2010)). More generally, history is rife with instances where means of payment have been subject to deceitful intent. The clipping of gold and silver coins in medieval Europe and rampant production of fake banknotes in the 19th century U.S. are notable examples, as documented in Mihm (2007) and recently formalized in Lester et al. (2012) and Li et al. (2012).

[5] The idea that what government accepts as payment affects what private agents do dates back to Smith (1963) and Lerner (1947). For example, Lerner (1947) argued that "the modern state can make anything it chooses generally acceptable as money… It is true that a simple declaration that such and such is money will not do… but if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done." Aiyagari and Wallace (1997) and Li and Wright (1998) provide the first formalizations of this insight.

[6] For instance in Head and Shi (2003) and Wright and Trejos (2001), there are no equilibria where a currency circulates only locally as sellers tend to accept all currencies. Without technical assumptions such as indivisible money and take-it-or-leave-it offers by buyers, all currencies tend to circulate as international currencies.

The study of international currency with search theory also follows a rich tradition, starting with the pioneering work of Matsuyama et al. (1993). Subsequent two-country, two-currency search models include contributions by Zhou (1997), Wright and Trejos (2001), Trejos (2003), Head and Shi (2003), Camera and Winkler (2003), Li and Matsui (2009), and Liu and Shi (2010).[7] However, these previous studies either place restrictions on asset divisibility or cannot generate the full class of acceptance patterns in this paper as equilibrium outcomes.

Regarding policy, a particularly related work is Li and Matsui (2009), who also study currency competition among welfare-maximizing monetary authorities, but must proxy for inflation in a model with indivisible currencies. Changes in the money supply will imply different distributions of money holdings, which make their policy implications not robust. In addition, terms of trade cannot adjust with the "inflation tax", which is a key channel for the transmission of monetary policy highlighted in this paper. For instance, the present paper shows how the emergence of an international currency, through changes in inflation, affects the terms of trade by expanding the set of trading opportunities. This generates important welfare gains from international currency use, which may not be fully captured in previous models with indivisible money.

A key contribution of this paper is to render the nominal exchange rate determinate in a setting where multiple currencies can coexist at potentially different rates of return. This is accomplished through assumptions on the economy's information frictions—namely anonymity and imperfect recognizability of currencies. Formally, this paper generalizes the model of asset liquidity by Lester et al. (2012) to an open-economy setting. While the authors discuss dollarization and exchange rates, having a multi-country model yields several additional contributions. When interactions between countries are explicitly modeled, it becomes possible to link how structural differences and heterogeneities across countries affect the circulation patterns that arise. Incorporating these heterogeneities also brings the model closer to more mainstream open-economy macro models, while still being explicit about the frictions that make money essential. Moreover, policymakers face tradeoffs in an open economy that do not appear in a closed economy. For instance, this paper shows that inflation can have redistributive effects across countries and that some inflation can be welfare-improving. Consequently, the policy implications of the present paper differs from that of Lester et al. (2012), where the Friedman rule is the unique optimal policy, so long as domestic currency is perfectly recognizable.

Liu and Shi (2010) also consider optimal monetary policy but focus on two symmetric currency areas. Further, agents in their model can hold either one of the two currencies, but not both. This restriction rules out equilibria where a subset of agents accepts both domestic and foreign currencies, a key focus of this paper. Moreover, since currency acceptability is not endogenous as in the present paper, inflation is not disciplined by the potential loss of international currency status.

Finally, while this paper assumes that sellers always reject assets they do not recognize, thereby simplifying the bargaining problem, an alternative formalization by Li et al. (2012) can provide stronger microfoundations for the information cost in this paper. This paper admittedly abstracts from informational asymmetries in the bargaining problem in order to focus on the international monetary issues highlighted above.

2. Environment

Time is discrete and continues forever. There are two countries, 1 and 2, populated with a continuum of 2 and $2n$ agents, respectively, where $n \in (0,1)$ denotes relative country size. Each period consists of two sub-periods: the first for decentralized trade in local and foreign goods and the second for settlement and currency exchange.

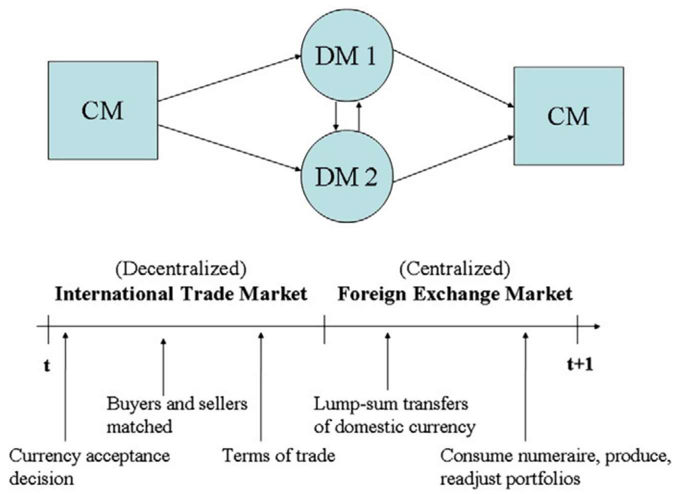

Fig. 1. Timing of representative period.

At the beginning of each period, agents first trade in decentralized markets (DM) of each country. Agents from each country are evenly divided between *buyers* and *sellers*: sellers from $s = \{1,2\}$ can produce output, q_s, but do not want to consume, while buyers want to consume but cannot produce. Sellers have immobile factors of production and cannot produce the other country's good. In the second sub-period, all trade occurs in a frictionless competitive market (CM). All agents can consume a numéraire good, x, which is produced according to a linear production function in labor. The supply of hours in the CM is h, which implies the real wage rate is equal to one. Fig. 1 summarizes the market structure and timing of events.

For tractability, instantaneous utilities for buyers and sellers are additively separable and quasi-linear in hours:

$$\mathcal{U}^B = u(q_s) + U(x) - h,$$
$$\mathcal{U}^S = -c(q_s) + U(x) - h.$$

To ease presentation, functional forms for utilities and cost functions are assumed to be the same across countries, though this assumption is relaxed in Appendix B. Further, $u(\cdot)$ and $c(\cdot)$ are assumed to be C^2 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$. Also, let $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$ and $x^* \in (0,\infty)$ solve $U'(x^*) = 1$. All goods are perishable, and agents discount the future between periods with a discount factor $\beta \in (0,1)$. Since agents lack commitment and individual histories are private information in the DM, unsecured credit cannot be used, which makes a medium of exchange essential for trade.

Each country issues its own fiat currency, $i = \{1,2\}$, both perfectly divisible and storable. Currency $m_i \in \mathbb{R}_+$ is valued at ϕ_i, the price of money in terms of the numéraire. The nominal exchange rate is defined here as the price of currency 2 in terms of currency 1: $e \equiv \frac{\phi_2}{\phi_1}$. Since market clearing in the CM implies that the law of one price holds, agents can trade currencies at the market clearing exchange rate. Hence, the CM also functions as a foreign exchange market. Money supplies, M_i, grow or shrink each period at a constant rate $(\gamma_i - 1)$, where $\gamma_i \equiv \frac{M_i'}{M_i}$. Variables with a prime denote next period's parameters or choices. Changes in the money supply are implemented through lump-sum monetary transfers or taxes of domestic currency in the CM to that country's buyers.[8]

[7] There are also one-country models that study currency substitution, such as Chang (1994), Uribe (1997), Engineer (2000), Ravikumar and Wallace (2002), Curtis and Waller (2000), Camera et al. (2004), and Martin (2006), among many others. See also Craig and Waller (2000) for a survey on dual-currency search models.

[8] When $\gamma_i < 1$, governments are assumed to have enough coercive power to collect taxes in the CM, but have no coercive power in the DM. Hu et al. (2009) and Andolfatto (2013) consider alternative formalizations where the buyers can choose not to participate the CM in order to avoid paying taxes. In the present model, this would generate a lower bound on the deflation rate that makes the Friedman rule infeasible.

Agents meet pairwise and at random in the DM where search and information frictions make international trade more difficult than local trade. Buyers are mobile while sellers are immobile. With probability $\alpha \in [\frac{1}{2}, 1]$, a buyer stays in his country of origin and with probability $1-\alpha$, visits the foreign country. The number of trade matches in the DM of country j is given by the matching function $\mathcal{M}_j \equiv \mathcal{M}(\mathcal{B}_j, \mathcal{S}_j) = \frac{\mathcal{B}_j \mathcal{S}_j}{\mathcal{B}_j + \mathcal{S}_j}$, where $\mathcal{B}_j$ and $\mathcal{S}_j$ denote the measures of buyers and sellers in the DM of country j. In country 1, $\mathcal{B}_1 = \alpha + n(1-\alpha)$, $\mathcal{S}_1 = 1$, and conditional on being in country 1, a buyer meets a seller with probability $a_1 \equiv \frac{\mathcal{M}_1}{\mathcal{B}_1} = \frac{1}{1+\alpha+n(1-\alpha)}$ while in country 2, $\mathcal{B}_2 = \alpha n + 1 - \alpha$, $\mathcal{S}_2 = n$, and conditional on being in country 2, a buyer meets a seller with probability $a_2 \equiv \frac{\mathcal{M}_2}{\mathcal{B}_2} = \frac{1}{1+\alpha+\frac{1-\alpha}{n}}$. Similarly, the probability a seller from j meets any buyer is $\frac{\mathcal{M}_j}{\mathcal{S}_j} = a_j \frac{\mathcal{B}_j}{\mathcal{S}_j}$, and the probability a seller from country j meets a buyer from i is $\lambda_{ij} = \mu_{ij} \frac{\mathcal{B}_j}{\mathcal{S}_j}$, where μ_{ij} represents the probability a buyer from i meets a seller from j given by the entries in Table 1.[9]

National and international trade frictions are controlled by the parameters n and α. Since $n \in (0,1)$, international meeting probabilities differ across countries. α can be interpreted as the degree of economic integration: as $\alpha \to \frac{1}{2}$, countries become more integrated and meeting a foreigner is more likely, while $\alpha \to 1$ corresponds to a closed economy where only locals trade.

Sellers in the model are further split between *private sellers*, who can choose which currencies to accept, and *government sellers*, who follow exogenous acceptance policies. While domestic currency is perfectly recognizable, it is difficult for private sellers to verify the quality of foreign currency. In particular, they must incur a fixed flow cost, $\psi_s \geq 0$, in order to recognize and hence accept payment in foreign money. For example, firms must invest in a verification device in order to authenticate genuine foreign notes from counterfeits.[10] The fixed cost is homogenous across sellers within a country but can differ across countries. It is common knowledge in a match whether the seller has invested, and sellers do not accept currencies they do not recognize. For example, sellers will reject payment if it is costless to produce worthless counterfeits: if sellers accepted unrecognizable currencies, buyers would just hand over counterfeits in each exchange. This assumption simplifies the pricing mechanism, as emphasized in Rocheteau (2008). Hence trade occurs under full information, and both currencies are accepted if and only if ψ_s is incurred.

Government sellers consume and produce just like private sellers and are subject to the same constraints and matching technology, but have exogenous policies regarding what they accept as payment. Governments that only accept its domestic currency is considered the baseline policy, τ. Government behavior is modeled this way since the purpose is to make precise how the size and influence of government affect realms of circulation and the set of equilibria. Given τ, terms of trade are determined through the same pricing mechanism as with interactions with private sellers.

Table 2 summarizes the sizes and composition of the world economy. Agents are equally split between buyers and sellers, each of size 1 and n in countries 1 and 2, respectively. The fraction of government sellers in each country, $g_1 \in [0,1]$ and $g_2 \in [0,1]$, can be interpreted as government size or the degree of centralized control.

Table 1
Buyer's meeting probabilities.

	Seller from 1	Seller from 2
Buyer from 1	αa_1	$(1-\alpha)a_2$
Buyer from 2	$(1-\alpha)a_1$	αa_2

3. Equilibrium

This section describes the equilibrium of the two-country, two-currency model. The focus is on a stationary equilibrium where aggregate real balances in each country are constant over time. Therefore, the rate of return of currency i in each country is constant and will equal $\gamma_i^{-1} = \frac{\phi_i'}{\phi_i}$.

3.1. Centralized market (CM) value functions

In the centralized market, a representative buyer of each country chooses consumption of the numéraire good x, labor h, and real balances to bring forward next period. Portfolios are expressed in real terms: let $\mathbf{z} \equiv (z_1, z_2) \equiv (\phi_1 m_1, \phi_2 m_2) \in \mathbb{R}_+^2$ represent a buyer from 1's portfolio of assets, and let $\hat{\mathbf{z}} \equiv (\hat{z}_1, \hat{z}_2) \equiv (\phi_1 \hat{m}_1, \phi_2 \hat{m}_2) \in \mathbb{R}_+^2$ denote a buyer from 2's portfolio. Also let $W_1^B(\mathbf{z})$ and $V_1^B(\mathbf{z})$ ($W_2^B(\hat{\mathbf{z}})$ and $V_2^B(\hat{\mathbf{z}})$) denote value functions for buyers from 1 (2) in the CM and DM, respectively.

In the beginning of the CM, a buyer from country 1 faces the following maximization problem:

$$W_1^B(\mathbf{z}) = \max_{x,h,\mathbf{z}' \in \mathbb{R}_+^2} \left\{ U(x) - h + \beta V_1^B(\mathbf{z}') \right\} \tag{1}$$

$$\text{s.t.} \quad x + \phi_1 m_1' + \phi_2 m_2' = h + z_1 + z_2 + T_1 \tag{2}$$

$$T_1 \equiv (\gamma_1 - 1)\phi_1 M_1. \tag{3}$$

The portfolio taken into the next DM is $\mathbf{z}' = (z_1', z_2') = (\phi_1' m_1', \phi_2' m_2')$, while T_1 is the lump-sum transfer of domestic currency from the government (expressed in numéraire goods). Substituting $m_c' = \frac{z_c'}{\phi_c'}$ for currency $c = \{1,2\}$ into the budget constraint and then eliminating h yields

$$W_1^B(\mathbf{z}) = U(x^*) - x^* + z_1 + z_2 + T_1 + \max_{\mathbf{z}' \in \mathbb{R}_+^2} \left\{ -\gamma_1 z_1' - \gamma_2 z_2' + \beta V_1^B(\mathbf{z}') \right\}. \tag{4}$$

A buyer from 1's lifetime utility at the beginning of the CM is the sum of his net consumption in the CM, real balances in domestic and foreign currency, the lump-sum transfer from the local government, and the continuation value at the beginning of the next DM minus

Table 2
Measures of agents in economy.

	Country 1	Country 2
Buyer	1	n
Private seller	$1-g_1$	$n(1-g_2)$
Government	g_1	ng_2
Total	2	$2n$

[9] The matching function satisfies the technical properties found in Berentsen et al. (2007) which specifies a general version of this matching function. Appendix A provides additional details for calculating the meeting probabilities for buyers and sellers.

[10] More broadly, there are a myriad of costs associated with adopting multiple means of payment, such as installing new technologies, dealing with foreign exchange traders, and learning how to use foreign currency. See also Chang (1994), Engineer (2000), and Martin (2006) for related formalizations. In a similar vein, Okawa and van Wincoop (2012) assume that due to differences in language and regulatory systems or easier access to local information, domestic agents are more informed than foreigners about domestic equity claims.

the investment in real balances. The value function for a buyer from 2 will be similar.

A few results from the CM value function are worth highlighting. First, $W_1^B(\mathbf{z})$ is linear in total wealth $\bar{z} = z_1 + z_2$: $W_1'(\bar{z}) = 1$. Second, there are no wealth effects since z' is independent of z, which follows from the quasi-linearity of the utility function. So long as holding money is costly, $\gamma_c \geq \beta$, sellers have no strict incentive to carry real balances in the DM, and they will leave the CM with no asset holdings. Consequently, their CM value function can be written as $W_i^S(\mathbf{z}) = U(x^*) - x^* + z_i + z_j + \beta V_i^S(0)$, which is also linear in total wealth.

Provided that DM value functions are strictly concave, there will generally be a unique portfolio where all buyers in a country demand the same real balances. A caveat is when the two currencies are perfect substitutes; in that case, buyers can hold different portfolios but they will have the same total value. I next specify the pricing mechanism in the DM, which will guarantee the strict concavity of the DM value functions.

3.2. Terms of trade

Terms of trade in the DM are determined according to Kalai's (1977) proportional bargaining rule. This pricing mechanism permits sellers to extract a constant fraction of the match surplus in order to recover some of their ex-ante investment.[11] Under proportional bargaining, a buyer acquires output in exchange for payment to the seller and receives a constant share, $\frac{\theta}{1-\theta}$, of the seller's surplus, where $\theta \in (0,1]$ measures the buyer's bargaining power, and threat points are given by continuation values.[12] Let q_s, d_s (Q_s,D_s) denote output and payment in private (government) meetings.

Given the model specification, terms of trade will depend on buyers' portfolios, private sellers' acceptance strategy, and governments' transaction policy. To apply the pricing mechanism, notice that the surplus of a buyer who gets q_s for payment d_s to a private seller is $u(q_s) + W_i^B(\bar{z} - d_s) - W_i^B(\bar{z}) = u(q_s) - d_s$, by the linearity of W_i^B. Similarly, the seller's surplus is $d_s - c(q_s)$.

Consider first a meeting between a buyer from 1 and a seller from s who only accepts domestic currency. Under proportional bargaining, quantity traded and payment to the seller, (q_s,d_s) solves

$$\max_{q_s,d_s} [u(q_s) - d_s] \tag{5}$$

$$\text{s.t.}\ \ u(q_s) - d_s = \frac{\theta}{1-\theta}[d_s - c(q_s)] \tag{6}$$

$$d_s \leq z_s. \tag{7}$$

The bargaining problem maximizes the buyer's surplus, subject to each party receiving a constant share of the match surplus, and a feasibility constraint (Eq. 7) that says the buyer cannot transfer more real balances than he current holds, which is just real balances in the seller's domestic currency, z_s. Consequently, the bargaining problem must satisfy

$$\begin{aligned} & q_s \in \arg\max \theta\ \ [u(q_s) - c(q_s)] \\ \text{s.t.}\ \ & d_s = (1-\theta)u(q_s) + \theta c(q_s) \leq z_s. \end{aligned}$$

The transfer of wealth from buyers to sellers is therefore given by the non-linear function $p(q_s) \equiv \theta c(q_s) + (1-\theta)u(q_s)$. As a result, output q_s solves

$$p(q_s) = \min\{p(q^*), z_s\} \tag{8}$$

where

$$p(q_s) \equiv \theta c(q_s) + (1-\theta)u(q_s). \tag{9}$$

The bargaining solution simply says that when $z_s \geq p(q^*)$, the buyer has enough wealth to finance purchase of the first-best q^*, and payment to the seller will be $p(q^*) = \theta c(q^*) + (1-\theta)u(q^*)$. When $z_s < p(q^*)$, the buyer just gives the seller his cash on hand, z_s, and gets in return $q_s < q^*$.

When instead the seller accepts both currencies by incurring the fixed cost to recognize foreign money, terms of trade will satisfy a similar problem as Eqs. (5)–(6), but with the feasibility constraint $d_s^b \leq z_1 + z_2$ since the buyer can now pay with both currencies. In what follows, the superscript b is used to distinguish variables when sellers accept both currencies. Consequently, output q_s^b solves

$$p\left(q_s^b\right) = \min\{p(q^*), z_1 + z_2\} \tag{10}$$

where

$$p\left(q_s^b\right) \equiv \theta c\left(q_s^b\right) + (1-\theta)u\left(q_s^b\right). \tag{11}$$

Since terms of trade with government sellers follow the same bargaining protocol as with private sellers, output in government meetings, $Q_s(\tau)$, will satisfy similar expressions:

$$p(Q_s(\tau)) = \min\{p(Q^*), z_s\} \tag{12}$$

where

$$p(Q_s(\tau)) \equiv \theta c(Q_s(\tau)) + (1-\theta)u(Q_s(\tau)). \tag{13}$$

3.3. Foreign currency acceptance decision

Before matches are formed in the DM, private sellers can acquire at some cost the information, or technology, in order to accept payment in foreign currency. Since the decision to acquire information about a currency determines its acceptability in trade, this breaks the indeterminacy that results when multiple assets can be used as means of payment.[13]

[11] Proportional bargaining ensures that trade is pairwise Pareto efficient and has several desirable features that cannot be guaranteed with Nash bargaining, as discussed in Aruoba et al. (2007). First, it guarantees the concavity of agents' value functions. Second, the proportional solution is monotonic and hence does not suffer from a shortcoming of Nash bargaining that an agent can end up with a lower individual surplus even if the size of the total surplus increases. Note that if the buyer makes a take-it-or-leave-it offer, sellers will have no incentive to incur the fixed cost to accept currencies since they do not receive any surplus from trade. There are also strategic foundations for the proportional bargaining solution. For instance in Dutta (2012), Kalai's (1977) solution emerges as a unique steady-state equilibrium outcome in a limiting case of a Nash demand game.

[12] Appendix B allows bargaining power to differ across countries and shows how differences in market structure affect the internationalization of currencies. For instance, as the bargaining power of a seller from s falls, equilibria where the seller's domestic currency is international become more likely while equilibria where the other country's currency is international become less so. See also Appendix B for a discussion on the importance of the economy's market structure or pricing mechanism on the type of equilibria that emerge.

[13] Without this decision, the model would predict that the two currencies are perfect substitutes that circulate at the same rate of return. Consequently, the composition of portfolios and the nominal exchange rate, $e = \frac{\phi_2}{\phi_1}$, is indeterminate, as in Kareken and Wallace (1981). This indeterminacy however is sensitive to the pricing mechanism. An alternative approach such as the Zhu and Wallace (2007) solution that treats domestic and foreign currencies asymmetrically can determine the exchange rate, as shown in Nosal and Rocheteau (2011).

Sellers' strategies are given by $\sigma_s \in [0,1]$, where $\sigma_s = 0$ if a private seller from $s = \{1,2\}$ rejects payment in foreign currency and $\sigma_s = 1$ if foreign currency is accepted. When $\sigma_s \in (0,1)$, both currencies are accepted in a fraction of trades. Due to the presence of government sellers that always accept local currency, there will be residual demand for both currencies. Without government sellers, there may also be a strategy where private sellers only accept foreign currency or reject payment altogether. Government transaction policies thus set a natural anchor for the acceptability of local money.

Given the bargaining solution, a private seller's expected payoff if he rejects payment in foreign currency is

$$\Pi_s \equiv (1-\theta)\{\lambda_{1s}[u(q_s)-c(q_s)] + \lambda_{2s}[u(\hat{q}_s)-c(\hat{q}_s)]\}$$

where λ_{is} denotes the probability that a seller from $s = \{1,2\}$ meets a buyer from $i = \{1,2\}$, $(1-\theta)$ is the seller's share in the trade surplus, and $\hat{q}_s$ is the DM output in meetings with country 2 buyers.

If instead a private seller incurs the fixed cost to accept foreign money, his expected payoff is

$$\Pi_s^b \equiv -\psi_s + (1-\theta)\left\{\lambda_{1s}\left[u\left(q_s^b\right)-c\left(q_s^b\right)\right] + \lambda_{2s}\left[u\left(\hat{q}_s^b\right)-c\left(\hat{q}_s^b\right)\right]\right\}.$$

A private seller's expected gain from accepting both currencies is therefore

$$\begin{aligned}\Delta_s &\equiv \Pi_s^b - \Pi_s \\ &= -\psi_s + (1-\theta)\left\{\lambda_{1s}\left[S\left(q_s^b\right)-S(q_s)\right] + \lambda_{2s}\left[S\left(\hat{q}_s^b\right)-S(\hat{q}_s)\right]\right\},\end{aligned}$$

where $S(\cdot) \equiv u(\cdot) - c(\cdot)$ defines total surplus in DM trades.

Consequently, the seller will choose to invest if $\Delta_s > 0$ and not invest if $\Delta_s < 0$. When $\Delta_s = 0$, sellers are indifferent and invest with an arbitrary probability. Optimal strategies $\sigma = (\sigma_1, \sigma_2)$ must therefore satisfy

$$\sigma_s = \begin{cases} 1 \\ \in [0,1] \\ 0 \end{cases} \text{if } \Delta_s \begin{cases} > \\ = 0. \\ < \end{cases} \tag{14}$$

3.4. Decentralized markets (DM) value function

Given the bargaining solution, the DM value functions simplify greatly. Since the terms of trade do not depend on sellers' portfolios, the DM value function can be written solely in terms of the buyer's problem.

Consider a representative buyer from country 1. Using the linearity of $W_1^B(z)$ and the bargaining solution, the DM value function simplifies to:

$$\begin{aligned}V_1^B(\mathbf{z}) = \alpha a_1 &\underbrace{\left\{(1-g_1)\theta\left[\sigma_1 S\left(q_1^b\right) + (1-\sigma_1)S(q_1)\right] + g_1\theta S(Q_1(\tau))\right\}}_{\text{buyer's surplus in domestic meetings}} \\ &+ (1-\alpha)a_2 \underbrace{\left\{(1-g_2)\theta\left[\sigma_2 S\left(q_2^b\right) + (1-\sigma_2)S(q_2)\right] + g_2\theta S(Q_2(\tau))\right\}}_{\text{buyer's surplus in foreign meetings}} \\ &+ z_1 + z_2 + W_1^B(0).\end{aligned}$$

The last three terms result from the linearity of $W_1^B(\mathbf{z})$ and is the value of proceeding to the CM with one's portfolio intact. A country 2 buyer's value function can be obtained by reversing the subscripts, replacing z with $\hat{z}$, and replacing q with $\hat{q}$.

Next, lead the DM value function forward by one period and substitute into the CM value function to yield the buyer's objective function. Letting the interest rate on an illiquid nominal bond denominated in currency c be $1 + i_c = \frac{\phi_c}{\phi_c' \beta}$, the buyer's optimal choice of real balances solves

$$\begin{aligned}\max_{\mathbf{z} \in \mathbb{R}_+^2}\{ &\underbrace{-i_1 z_1 - i_2 z_2}_{\text{cost of holding money}} + \alpha a_1 \underbrace{\left\{(1-g_1)\theta\left[\sigma_1 S\left(q_1^b\right) + (1-\sigma_1)S(q_1)\right] + g_1\theta S(Q_1(\tau))\right\}}_{\text{buyer's surplus in domestic meetings}} \\ &+ (1-\alpha)a_2 \underbrace{\left\{(1-g_2)\theta\left[\sigma_2 S\left(q_2^b\right) + (1-\sigma_2)S(q_2)\right] + (1-g_2)\theta S(Q_2(\tau))\right\}}_{\text{buyer's surplus in foreign meetings.}}.\end{aligned} \tag{15}$$

Here $i_c = \frac{\phi_c}{\phi_c' \beta} - 1$ is the cost of holding currency c. The objective function simply says that a buyer chooses a portfolio to maximize his expected surplus in domestic and foreign meetings, net of the cost of holding currency. Since Eq. (15) is continuous and maximizes over a compact set, a solution to the buyer's problem exists. Appendix A verifies the strict concavity of the buyer's problem and derives the corresponding first-order necessary (and sufficient) conditions.

Definition 1. Given τ, a stationary monetary equilibrium is a list of quantities traded $\left\{\left(q_s, q_s^b, Q_s(\tau)\right), \left(\hat{q}_s, \hat{q}_s^b, \hat{Q}_s(\tau)\right)\right\}$, sellers' strategies σ_s, and real balances $\{\mathbf{z} \equiv (z_1, z_2), \hat{\mathbf{z}} \equiv (\hat{z}_1, \hat{z}_2)\} \forall s = \{1,2\}$ such that

1. $\left\{\left(q_s, q_s^b, Q_s(\tau)\right), \left(\hat{q}_s, \hat{q}_s^b, \hat{Q}_s(\tau)\right)\right\} \in \mathbb{R}_+^3 \times \mathbb{R}_+^3$ solves the bargaining problem;
2. $\sigma_s \in [0,1]$ solves sellers' currency acceptance decision;
3. $\{(z_1, z_2), (\hat{z}_1, \hat{z}_2)\} \in \mathbb{R}_+^2 \times \mathbb{R}_+^2$ solves buyers' portfolio problem;
4. Money markets clear.

In a monetary equilibrium where both currencies are valued, $z_1 > 0$ and $z_2 > 0$, DM output must satisfy

$$\begin{aligned}i_1 = \alpha a_1 &\left\{(1-g_1)\left[\sigma_1 L\left(q_1^b\right) + (1-\sigma_1)L(q_1)\right] + g_1 L(Q_1(\tau))\right\} \\ &+ (1-\alpha)a_2(1-g_2)\left[\sigma_2 L\left(q_2^b\right)\right]\end{aligned} \tag{16}$$

$$\begin{aligned}i_2 = \alpha a_1 &(1-g_1)\left[\sigma_1 L\left(q_1^b\right)\right] \\ &+ (1-\alpha)a_2\left\{(1-g_2)\left[\sigma_2 L\left(q_2^b\right) + (1-\sigma_2)L(q_2)\right] + g_2 L(Q_2(\tau))\right\}\end{aligned} \tag{17}$$

where

$$L(\cdot) \equiv \frac{\theta[u'(\cdot) - c'(\cdot)]}{\theta c'(\cdot) + (1-\theta)u'(\cdot)}.$$

Buyers wish to bring currencies into the DM since these objects facilitate trade across different meeting types, but doing so is costly as captured by the terms i_1 and i_2 on the left sides of Eqs. (16) and (17). The function $L(\cdot)$ is the *liquidity premium* and represents the marginal payoff an agent gets from his liquid wealth that can be used to acquire more output in the DM instead of carrying it over to the subsequent CM. There are analogous equations for buyers in country 2: for $\hat{z}_1 > 0$ and $\hat{z}_2 > 0$, DM output must satisfy

$$\begin{aligned}i_1 &= (1-\alpha)a_1\left\{(1-g_1)\left[\sigma_1 L\left(\hat{q}_1^b\right) + (1-\sigma_1)L(\hat{q}_1)\right] + g_1 L\left(\hat{Q}_1(\tau)\right)\right\} \\ &\quad + \alpha a_2(1-g_2)\left[\sigma_2 L\left(\hat{q}_2^b\right)\right], \\ i_2 &= (1-\alpha)a_1(1-g_1)\left[\sigma_1 L\left(\hat{q}_1^b\right)\right] + \alpha a_2\left\{(1-g_2)\left[\sigma_2 L\left(\hat{q}_2^b\right) + (1-\sigma_2)L(\hat{q}_2)\right]\right. \\ &\quad \left. + g_2 L\left(\hat{Q}_2(\tau)\right)\right\}.\end{aligned}$$

Intuitively, the equilibrium conditions equate the marginal benefit of liquidity to its cost. As a result, a currency demands a liquidity premium only if it is accepted in trade, as determined by σ_s and τ. When no sellers accept a currency, it will not be valued. Also notice that $L(\cdot)$ is strictly decreasing in DM output over the relevant range: that is, $L'(\cdot) < 0$ for

Table 3
Equilibrium currency regimes.

Regime	σ	Circulation pattern
N	(0,0)	Two national currencies
I_1	(0,σ_2)	Currency 1 is international and currency 2 is national
I_2	(σ_1,0)	Currency 2 is international and currency 1 is national
U	(1,1)	Two international currencies

$q \in [0,q^*]$. In what follows, the focus is on equilibria where $\gamma_i \geq \beta$, since there is no solution otherwise.

The following lemma summarizes some basic properties of optimal portfolio holdings.

Lemma 1. *Consider any stationary monetary equilibrium where $i_1 \neq i_2$ (currencies are not perfect substitutes).*

1. *All buyers from the same country hold the same portfolios.*
2. *Buyers from different countries will generally hold different portfolios.*
3. *When there are no asymmetries in meeting arrangements—i.e., when the economy is perfectly integrated ($\alpha = \frac{1}{2}$) and both countries and governments are of equal sizes ($n = 1$, $g_1 = g_2$)—then all buyers, irrespective of country origin, will hold the same portfolios.*

The intuition of Lemma 1 is that buyers from different countries hold different portfolios due to the asymmetry in the matching process: since the probability of meeting a foreigner depends on one's nationality, buyers allocate portfolio weights accordingly. Without any asymmetry in meeting arrangements, then buyers' nationalities cease to matter and will all hold the same portfolios. Given the model specification, this requires that $n = 1$, $g_1 = g_2$, and $\alpha = \frac{1}{2}$, which implies that it is equally likely to meet compatriots as foreigners.

Finally to close the model, market clearing implies that for each currency, aggregate supply must equal aggregate demand. By Lemma 1, all buyers from the same country hold the same portfolio when currencies are not perfect substitutes. Total demand for money 1 is $m_1 + n\hat{m}_1$ and for money 2 is $m_2 + n\hat{m}_2$. Market clearing then implies

$$m_1 + n\hat{m}_1 = M_1, \tag{18}$$

$$m_2 + n\hat{m}_2 = M_2. \tag{19}$$

4. Currency regimes

Having defined monetary equilibrium, I now examine the types of currency regimes that arise in the dual-currency economy. Given government transaction policies under τ, a currency regime is defined as a pair of strategies for private sellers, $\sigma \equiv (\sigma_1,\sigma_2)$, that satisfies their currency acceptance decision. The focus is on the most representative monetary regimes: local circulation of currencies and international circulation of one or both currencies. Table 3 summarizes the currency regimes discussed in the text. In the following, the implications and existence of these types of equilibria are discussed. Finally it is implicit that all endogenous variables in a particular equilibrium are indexed with the regime under consideration.

4.1. Regime N: two national currencies

Consider first a regime where sellers only accept their domestic currency; that is, $\sigma = (0,0)$. Suppose that country 1 is the U.S. and country 2 is Mexico. Given the government transaction policy τ, this gives rise to the emergence of two national currencies: dollars are only accepted in the United States and pesos are only accepted in Mexico.[14] This coincides with a common assumption in many international macroeconomic models, though arises as an equilibrium outcome in this model.

Under τ, output for country 1 buyers must satisfy

$$i_1 = \alpha a_1 L(q_1) \tag{20}$$

$$i_2 = (1-\alpha) a_2 L(q_2) \tag{21}$$

since output in government meetings will be the same as in private meetings (output for country 2 buyers will satisfy similar equations). Eqs. (20) and (21) relate the demand by agents for the two currencies to the cost of holding them. Since DM quantities in each meeting type can be obtained independently from one another, monetary policies are independent across countries. Real balances can then be obtained from the bargaining solution: $\phi_1 m_1 = p(q_1)$ and $\phi_2 m_2 = p(q_2)$ in country 1 and $\phi_1 \hat{m}_1 = p(\hat{q}_1)$ and $\phi_2 \hat{m}_2 = p(\hat{q}_2)$ in country 2. Necessary conditions for the two currencies to admit interior solutions are $i_1 < \bar{i}_1 \equiv \alpha a_1 \frac{\theta}{1-\theta}$ and $i_2 < \bar{i}_2 \equiv (1-\alpha) a_2 \frac{\theta}{1-\theta}$. Alternatively, buyers must receive enough of the gains from trade in order for currencies to be valued. Hence even with the presence of government sellers that always accept local currency, there may be a non-monetary equilibrium where neither currencies are valued if $i_1 > \bar{i}_1$ or $i_2 > \bar{i}_2$.

So long as $i_1 > \bar{i}_1$ or $i_2 > \bar{i}_2$, buyers hold positive balances of both monies since each has exchange roles in the issuing country.[15] Both currencies can be valued since each is essential for some meetings, even if one is being issued at a higher rate and thus has a lower rate of return. Hence low-return currencies can circulate in equilibrium despite the existence of a competing, higher-return currency. Since sellers only accept their local money, only national currencies change hands. Equilibrium money holdings can then be obtained through the market-clearing conditions, $m_1 + n\hat{m}_1 = M_1$ and $m_2 + n\hat{m}_2 = M_2$.

Turning to existence, this regime will constitute an equilibrium so long as private sellers have no incentive to incur the cost to recognize the foreign currency. This is true if $\prod_s > \prod_s^b$, or $\Delta_s < 0 \forall_s = \{1,2\}$, which is satisfied when ψ_1 and ψ_2 are large enough:

$$\psi_1 > \bar{\psi}_1 \equiv (1-\theta)\left\{\lambda_{11}\left[S\left(q_1^b\right) - S(q_1)\right] + \lambda_{21}\left[S\left(\hat{q}_1^b\right) - S(\hat{q}_1)\right]\right\},$$
$$\psi_2 > \bar{\psi}_2 \equiv (1-\theta)\left\{\lambda_{12}\left[S\left(q_2^b\right) - S(q_2)\right] + \lambda_{22}\left[S\left(\hat{q}_2^b\right) - S(\hat{q}_2)\right]\right\},$$

where λ_{is} is the probability a seller from s meets a country i buyer. Consequently, an equilibrium with national currencies will exist so long as both currencies are valued $\left(i_1 < \bar{i}_1 \text{ and } i_2 < \bar{i}_2\right)$ and private sellers only accept domestic currency ($\psi_1 > \bar{\psi}_1$ and $\psi_2 > \bar{\psi}_2$).

Since in this equilibrium, private sellers and government sellers have the same acceptance strategy, all matches with a given country's buyer are identical. This is reflected in the equilibrium conditions (20) and (21) which are independent of g_1 and g_2. Consequently, governments that only accept domestic currency are neither a necessary nor a

[14] A national currencies equilibrium is difficult to obtain in earlier dual-currency search models, as pointed out by Yiting Li. That a national currencies equilibrium arises endogenously is a novel contribution of this paper, an outcome made possible due to the imperfect recognizability friction.

[15] A buyer from the U.S. holds both monies since they may meet a Mexican that only accepts pesos, which generates a precautionary demand for foreign currency. Since this allows trade to occur between agents from different countries—though only the seller's domestic currency changes hands—this differs from the "autarky" regime discussed in Matsuyama et al. (1993). In the present model, only in a closed economy ($\alpha = 1$) will there be no international trade and hence no precautionary demand for foreign currency. Appendix B considers an alternative formalization of the model where buyers know with whom they will be matched with before making portfolio decisions. In that case, there is no longer a precautionary demand for foreign currency. Moreover, the set of equilibria from the baseline model is preserved in this alternative specification.

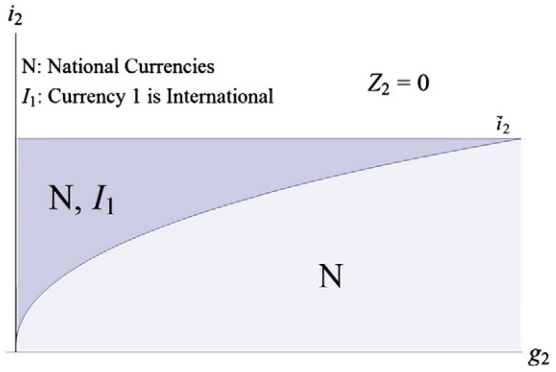

Fig. 2. Inflation and government policy.

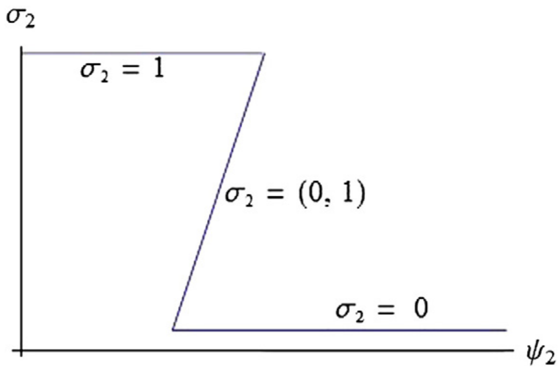

Fig. 3. Sellers' strategies and multiplicity.

sufficient condition for the existence of a national currency equilibrium. So long as both currencies are valued and private sellers find it optimal to reject foreign money, a national currency equilibrium will exist.

4.2. Regime I_1 and I_2: one international currency and one national currency

This class of equilibria corresponds to the emergence of an international currency that is accepted both locally and abroad. Within this class, there will be an equilibrium in pure strategies where all the U.S. sellers reject pesos while Mexican private sellers accept both dollars and pesos: $(\sigma_1,\sigma_2) = (0,1)$, as well as a mixed strategy equilibrium where Mexican private sellers randomize: $(\sigma_1,\sigma_2) = (0,\Phi_2)$, where $\Phi_2 \in (0,1)$. Another equilibrium, $(\sigma_1,\sigma_2) = \{(1,0),(\Phi_1,0)\}$ has symmetric properties as this regime, so discussion is omitted.

Under τ, output for country 1 buyers must satisfy

$$i_1 = \alpha a_1 L(q_1) + (1-\alpha)a_2(1-g_2)\sigma_2 L\left(q_2^b\right),$$
$$i_2 = (1-\alpha)a_2\left\{(1-g_2)\left[\sigma_2 L\left(q_2^b\right) + (1-\sigma_2)L(q_2)\right] + g_2 L(Q_2)\right\}.$$

Buyers hold positive balances of the two monies: $p(q_1) = \phi_1 m_1$, $p(q_2^b) = \phi_1 m_1 + \phi_2 m_2$, $p(q_2) = p(Q_2) = \phi_2 m_2$. Since the dollar is accepted by sellers from both countries, it emerges as an international medium of exchange. Symmetric conditions can be obtained for country 2 buyers.

The following table summarizes the effects of inflation and monetary policy in the two countries. Calculations are provided in Appendix A.

$\frac{\partial \phi_1}{\partial i_1} < 0$	$\frac{\partial \phi_2}{\partial i_1} > 0$	$\frac{\partial e}{\partial i_1} > 0$	$\frac{\partial \phi_1}{\partial g_1} = 0$	$\frac{\partial \phi_2}{\partial g_1} = 0$
$\frac{\partial \phi_1}{\partial i_2} > 0$	$\frac{\partial \phi_2}{\partial i_2} < 0$	$\frac{\partial e}{\partial i_2} < 0$	$\frac{\partial \phi_1}{\partial g_2} < 0$	$\frac{\partial \phi_2}{\partial g_2} > 0$

If γ_2 increases, the peso inflates, which decreases its value, ϕ_2. This also raises the value of dollars, ϕ_1. The exchange rate $e = \frac{\phi_2}{\phi_1}$ falls and dollars appreciate due to increased foreign demand. Intuitively, as domestic inflation increases, locals in Mexico economize on peso holdings, which reduces its price ϕ_2. Since there's less demand for pesos, agents substitute into dollars, which raises its price ϕ_1. Now that the dollar is more valuable, sellers have more incentive to accept it. As a result, the economy dollarizes. This is due to the model's general equilibrium effects that make currency substitution an endogenous response to local inflation. This situation arises precisely in dollarized economies where high inflation makes transacting in the local currency more costly so that citizens instead adopt a less inflationary asset such as the U.S. dollar.[16]

Fig. 2 illustrates the effect of inflation on transaction patterns. Consider a Mexican seller's decision to accept both currencies rather than just pesos. When peso inflation is low, the benefit of adopting an additional medium of exchange is also low. As monetary policy approaches the Friedman rule $i_2 \rightarrow 0$, output approaches q^*, the expected benefit of acquiring information Δ_2 gets small, and there is an equilibrium where sellers do not accept foreign money so that economy ends up in regime *N*. As inflation increases however, it becomes more costly to use local money, which decreases ϕ_2 and increases the value of the alternative asset, ϕ_1. This raises the incentive to acquire information and can generate multiple circulation patterns. As a result, currency substitution may arise as a purely expectational phenomenon. Historical episodes of dollarization in response to high inflation support this idea.[17]

Fig. 2 also shows how the government transaction policy variable, g_2, affects circulation patterns. Only in the limiting case where $g_2 = 1$ does the equilibrium where currency 1 is international cease to exist. For $g_2 < 1$, legal tender laws are therefore insufficient to rule out circulation of foreign currency. The non-monetary equilibrium when $i_2 > \bar{i}_2$ therefore exists even when enforcement is at its maximum, $g_2 = 1$. Fig. 2 also illustrates the possibility that a local currency may survive and coexist with an international medium of exchange even without government restrictions ($g_2 = 0$), as in Matsuyama et al. (1993). More generally, changes in government size may have direct effects on currency values, thereby influencing the monetary equilibrium attained.

Notice however that the government size of the issuing country does not affect the circulation of its currency nor the circulation of the competing currency. That is, $\frac{\partial \phi_1}{\partial g_1} = 0$ and $\frac{\partial \phi_2}{\partial g_1} = 0$ in this equilibrium. Since in country 1, private sellers follow the same acceptance strategies as government sellers, changes in g_1 cannot be used as a policy to affect the circulation of currency 1.

In this model, an international currency emerges due to sellers' acceptance decisions and becomes valued when a subset of sellers gets informed about both currencies. An equilibrium where currency

[16] Endogenous currency substitution also appears in Lester et al. (2012) with a fixed one-time cost in a closed-economy setting, as well as in other frameworks such as Chang (1994) and Martin's (2006) overlapping generation models, Uribe's (1997) cash-in-advance model, and Engineer's (2000) turnpike model.

[17] Currency substitution typically arises under high inflation, where the use of foreign currency persists after it has been accepted. For example, dollarization in many Latin American and Asian countries continues even after inflation stabilization, consistent with the model's predictions.

1 becomes international requires $\Pi_1 > \Pi_1^b$ and $\Pi_2 \leq \Pi_2^b$, which implies that the fixed cost is large enough in country 1 while low enough in country 2:

$$\psi_1 > \overline{\overline{\psi}}_1 \equiv (1-\theta)\left\{\lambda_{11}\left[S\left(q_1^b\right) - S(q_1)\right] + \lambda_{21}\left[S\left(\hat{q}_1^b\right) - S(\hat{q}_1)\right]\right\},$$
$$\psi_2 \leq \tilde{\psi}_2 \equiv (1-\theta)\left\{\lambda_{12}\left[S\left(q_2^b\right) - S(q_2)\right] + \lambda_{22}\left[S\left(\hat{q}_2^b\right) - S(\hat{q}_2)\right]\right\}.$$

For both currencies to be accepted, the flow cost for country 2 sellers must be less than the increase in their expected surplus associated with accepting both currencies. Fig. 3 depicts the strategy of a country 2 seller as a function of the fixed cost, ψ_2, and the measure of country 2 sellers that accept both currencies, σ_2. Since the two horizontal lines overlap for intermediate values of ψ_2, there can be multiple equilibria where regimes N and I_1 coexist.

4.3. Regime U: two international currencies

Now consider an equilibrium where all sellers accept both currencies, $\sigma = (1,1)$, leading to the emergence of two international currencies.[18]

The equilibrium condition is

$$i_1 = \alpha a_1 L\left(q_1^b\right) + (1-\alpha)a_2(1-g_2)L\left(q_2^b\right) + \alpha a_1 g_1 L(Q_1)$$
$$i_2 = \alpha a_1 L\left(q_1^b\right) + (1-\alpha)a_2(1-g_2)L\left(q_2^b\right) + (1-\alpha)a_2 g_2 L(Q_2).$$

In contrast with Lester et al. (2012), there can be endogenous liquidity differentials when all sellers accept both currencies due to heterogeneities across countries. That is, $i_1 = i_2$ is no longer a necessary condition for this type of equilibrium if countries are imperfectly integrated ($\alpha \neq \frac{1}{2}$), have different sizes ($n \neq 1$), or have governments of different sizes ($g_1 \neq g_2$).

If instead there is no government ($g_1 = g_2 = 0$), for both monies to be valued, it must be that

$$i_1 = i_2.$$

Hence the two currencies are equally liquid and are valued only if they have the same rate of return. Agents may hold different portfolios, but they will have the same total value: $p(q^b) = \phi_1 m_1 + \phi_2 m_2$ for country 1 buyers and $p\left(\hat{q}^b\right) = \phi_1 \hat{m}_1 + \phi_2 \hat{m}_2$ for country 2 buyers. While output q^b and $\hat{q}^b$ is uniquely pinned down, money holdings (m_1,m_2) and $(\hat{m}_1, \hat{m}_2)$ must satisfy the market-clearing conditions $m_1 + n\hat{m}_1 = M_1$ and $m_2 + n\hat{m}_2 = M_2$. With more unknowns than equations, this makes the exchange rate indeterminate as in Kareken and Wallace (1981).

While in Kareken and Wallace (1981) the nominal exchange rate is everywhere indeterminate, this is not the case in this model. In particular, the fixed costs of recognizing foreign currency constrain the indeterminacy.[19] If ψ_1 and ψ_2 are large enough so that no sellers accept both, there is no longer an equilibrium where the two currencies circulate at par. Conversely, an equilibrium where all private sellers accept both

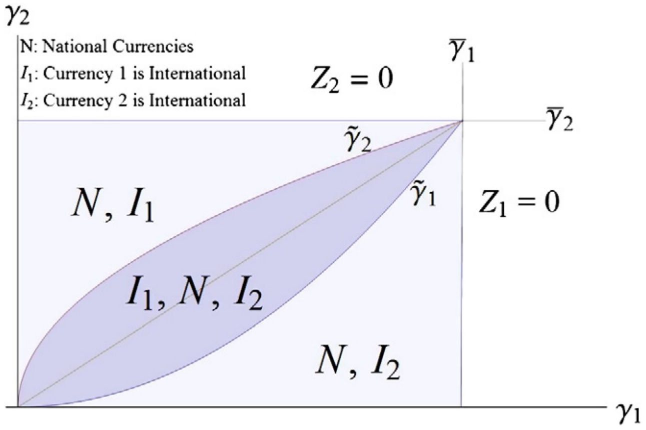

Fig. 4. Money growth rates: existence of equilibria in (γ_1,γ_2)-space.

currencies exists if $\Pi_s^b > \Pi_s \; \forall_s = \{1,2\}$, which implies that the fixed cost in both countries must be sufficiently low:

$$\psi_1 < \tilde{\tilde{\psi}}_1 \equiv (1-\theta)\left\{\lambda_{11}\left[S\left(q_1^b\right) - S(q_1)\right] + \lambda_{21}\left[S\left(\hat{q}_1^b\right) - S(\hat{q}_1)\right]\right\},$$
$$\psi_2 < \tilde{\tilde{\psi}}_2 \equiv (1-\theta)\left\{\lambda_{12}\left[S\left(q_2^b\right) - S(q_2)\right] + \lambda_{22}\left[S\left(\hat{q}_2^b\right) - S(\hat{q}_2)\right]\right\}.$$

While the first-best level of output $q^b = q^*$ is achieved under the Friedman Rule, this is not socially efficient since all sellers must incur a real cost $\psi_1 > 0$ and $\psi_2 > 0$.

4.4. Multiple equilibria

The model generates multiple equilibria where the share of transactions requiring different currencies is not uniquely determined by fundamentals. This multiplicity is illustrated in Fig. 4, which depicts the existence of equilibria as a function of the two country's money growth rates, (γ_1,γ_2), assuming that information costs are neither sufficiently high nor low so that Regimes N, I_1, I_2, and U are all possible.

An equilibrium with national currencies (regime N) exists so long as neither currency is too costly to hold: $\gamma_1 < \overline{\gamma}_1$ and $\gamma_2 < \overline{\gamma}_2$. An equilibrium where currency 1 is international while currency 2 is only locally accepted (regime I_1) exists so long as $\gamma_1 < \tilde{\gamma}_1$—the rate of return on currency 1 is high enough in order to give sellers from 2 enough incentive to accept currency 1. Symmetrically for regime I_2, an equilibrium where sellers accept both currencies (regime U) exists as a knife-edge case on the 45-degree line where there is rate of return equality. Finally, there can be a unique non-monetary equilibrium where either one or both currencies are not valued if monies are too costly to hold, which occurs when $\gamma_1 < \overline{\gamma}_1$ or $\gamma_2 < \overline{\gamma}_2$.

The intuition for multiplicity operates through the general equilibrium interaction between buyers and sellers: what sellers accept depend on what buyers carry, and what buyers carry depend on what sellers accept. When more sellers accept a currency, it becomes more liquid and thus more valuable in exchange. This makes buyers want to hold more, increasing its value further and increasing the incentives for sellers to accept it. Due to this complementarity, multiple equilibria can arise. Consequently, the regime that the economy ends up in will depend on both fundamentals and expectations regarding what other agents do.

4.5. Welfare

This section concludes with a discussion of the model's welfare properties. Due to the presence of multiple equilibria, countries may prefer one type of payment regime to another. Welfare in country $i \in \{1,2\}$ is defined as the steady-state sum of buyers' and

[18] There is also an equilibrium where sellers in both countries randomize, $\sigma = (\Phi_1,\Phi_2)$, where Φ_s denotes the fraction of sellers in country $s = \{1,2\}$ that accept both currencies. In this case, equilibrium conditions will be given by Eqs. (16) and (17), with $\sigma_1 = \Phi_1$ and $\sigma_2 = \Phi_2$. In general, the two monies need not be perfect substitutes.

[19] In addition, the presence of government sellers that only accept local currency also constrains exchange rate indeterminacy since there can still be liquidity differentials across currencies if governments are of different sizes ($g_1 \neq g_2$). An early analog of this idea is formalized in King et al. (1992), a two-currency overlapping generation model where a subset of agents is constrained to only accept a particular currency while the rest are free to demand payment in either currency or a combination of both.

sellers' utilities in country i, weighted by their respective measures in the DM, $\mathcal{B}_i$ and $\mathcal{S}_i$:

$$\mathcal{W}_i = \mathcal{B}_i(1-\beta)V_i^B + \mathcal{S}_i(1-\beta)V_i^S,$$

where net consumption in the CM, $U(x^*) - x^*$, is normalized to zero. Appendix A shows that welfare can be written as

$$\begin{aligned}\mathcal{W}_i = {} & \mathcal{B}_i\mathcal{T}_i + \alpha\mathcal{M}_i \underbrace{\left\{(1-g_i)\left[\sigma_i S\left(q_i^b\right) + (1-\sigma_i)S(q_i)\right] + g_i S(Q_i(\tau))\right\}}_{\text{total surplus in domestic meetings}} \\ & +(1-\alpha)\mathcal{M}_j\,\theta \underbrace{\left\{\left(1-g_j\right)\left[\sigma_j S\left(q_j^b\right) + \left(1-\sigma_j\right)S\left(q_j\right)\right] + g_j S\left(Q_j(\tau)\right)\right\}}_{\text{buyer } i\text{'s surplus in foreign meetings}} \\ & +(1-\alpha)\mathcal{M}_i \underbrace{(1-\theta)\left\{(1-g_i)\left[\sigma_i S\left(q_i^b\right) + (1-\sigma_i)S(q_i)\right] + g_i S(Q_i(\tau))\right\}}_{\text{seller } i\text{'s surplus in foreign meetings}} \\ & - \underbrace{(1-g_i)\mathcal{S}_i\sigma_i\psi_i}_{\text{private seller } i\text{'s information cost}},\end{aligned}$$

where

$$\mathcal{T}_1 \equiv \underbrace{n\phi_1(\hat{m}_1' - \hat{m}_1)}_{\text{country 1's seigniorage revenue from country 2}} - \underbrace{\phi_2(m_2' - m_2)}_{\text{country 1's seigniorage transfer to country 2}}, \tag{22}$$

$$\mathcal{T}_2 \equiv \underbrace{\phi_2(m_2' - m_2)}_{\text{country 2's seigniorage revenue from country 1}} - \underbrace{n\phi_1(\hat{m}_1' - \hat{m}_1)}_{\text{country 2's seigniorage transfer to country 1}}. \tag{23}$$

As a result, welfare can be decomposed into two components: (i) net seigniorage revenues or transfers given by $\mathcal{T}_i$, and (ii) surplus in DM trades net of information costs. In all equilibria where both currencies are valued, $\mathcal{T}_1 \neq 0$ and $\mathcal{T}_2 \neq 0$, with $\mathcal{T}_1 + \mathcal{T}_2 = 0$ due to market-clearing.

According to the theory, there are two distinct sources of the welfare benefits of having an international currency. First is the increase in welfare due to increased seigniorage that arises from increased demand for real balances by foreigners. Second is the change in welfare due to increased trade. When a currency becomes international, it is more widely used in facilitating transactions abroad, which expands international trade.

Consequently, the model implies that welfare is unambiguously higher for a country that successfully has its currency accepted abroad than under a national currency regime. This gain comes from two sources: seigniorage gains from foreigners and an expansion of trade opportunities. However whether the other country also benefits from foreign currency circulation is ambiguous and depends on the benefit from increased trade, the cost of lost seigniorage, and the cost of accepting foreign money. These tradeoffs will be especially important in the next section which considers a simple policy game to determine the choice of inflation in the dual-currency economy.

5. A simple monetary policy game

This section analyzes the strategic choices of monetary authorities by modeling their objective functions and specifying the rules of their strategic interaction.[20] In the baseline analysis, monetary authorities behave non-cooperatively and choose a money growth rate for its country to maximize the welfare of its citizens, taking as given the other country's money growth rate. In turn, each monetary authority is able to affect the rate of return of its currency and hence impact welfare both at home and abroad. Since the economy is open and policymakers behave strategically, the choice of inflation in one country depends not only on domestic transaction patterns, but also on choices made by the foreign policymaker and foreign citizens. As a result, policymakers may generate an externality for the other country that leads policy to deviate from the Friedman rule.

5.1. Social planner problem

As a benchmark to assess the efficiency properties of the policy game, consider the social planner's problem of choosing money growth rates for the two countries, (γ_1,γ_2), by maximizing total welfare for the world, $\mathcal{W} = \mathcal{W}_1 + \mathcal{W}_2$, subject to the frictions and resource constraints in the world economy.

Proposition 1. *The social planner's choice of money growth rates achieves the first-best level of output in all matches, q^*, and is attained by the joint Friedman rule in both countries, $(\gamma_1,\gamma_2) = (\beta,\beta)$. In that case, both currencies are valued and become unified as one money.*

5.2. Non-cooperative policy

This section starts by representing the strategic choices of monetary authorities as a one-shot non-cooperative game with perfect information. The analysis abstracts from repeated interactions among the monetary authorities since allowing for trigger strategies would substantially enlarge the set of equilibria.

There are three sets of players: citizens and the monetary authorities in countries 1 and 2. The game is divided into two stages. In the first stage, the monetary authority from each country $i = \{1,2\}$ chooses a money growth rate $\gamma_i \in [\beta,\infty)$ to maximize the welfare of its citizens, taking as given the money growth rate chosen by the other country, $j = \{1,2\} \neq i$, and optimal choices by citizens. Monetary authorities commit to their policy and choose their policies simultaneously and once-and-for-all. In the second stage, citizens observe the actions of monetary authorities, make their currency acceptance decision, settle terms of trade, and select portfolio holdings. Throughout this section, it is assumed that government sellers only accept domestic currency. The focus is on finding subgame perfect equilibria of the policy game.

Definition 2. *A subgame perfect equilibrium consists of money growth rates for monetary authorities, (γ_1^*,γ_2^*), and best response functions for agents, $\Theta^*(\gamma_1,\gamma_2)$, such that*

1. *For any given action taken by monetary authorities, agents' optimal choices, $\Theta^*(\gamma_1,\gamma_2) \equiv \left\{\left(q_s, q_s^b, Q_s(\tau)\right), \left(\hat{q}_s, \hat{q}_s^b, \hat{Q}_s(\tau)\right), (z_i, z_j), (\hat{z}_i, \hat{z}_j), \sigma_s\right\}$ satisfy Definition 1 for all $i,j,s \in \{1,2\}$, $i \neq j$.*
2. *Monetary authority $i \in \{1,2\}$ chooses a money growth rate, $\gamma_i^* \in [\beta,\infty)$, that maximizes welfare for its citizens, $\mathcal{W}_i$, taking as given γ_j^* for $j \in \{1,2\} \neq i$ and $\Theta^*(\gamma_1,\gamma_2)$:*

$$\begin{aligned}\gamma_1^* &\in \arg\max\ \mathcal{W}_1(\gamma_1, \gamma_2^*, \Theta^*(\gamma_1, \gamma_2^*)), \\ \gamma_2^* &\in \arg\max\ \mathcal{W}_2(\gamma_1^*, \gamma_2, \Theta^*(\gamma_1^*, \gamma_2)).\end{aligned}$$

The game is solved using backwards induction, starting with the choices made by citizens. Section 4 solved for these optimal choices by characterizing the currency regimes that emerge, which forms the Nash equilibria of the final subgame.

[20] Policy games in two-country, two-currency search models with indivisible money include Trejos and Wright (1996), Trejos (2003), and Li and Matsui (2009). While some of the tradeoffs faced by monetary authorities highlighted here also appear in Li and Matsui (2009), the transmission of monetary policy in the present paper occurs through terms of trade and welfare, which is notably absent in Li and Matsui (2009). Liu and Shi (2010) also consider optimal monetary policy but focuses on the deviations from the law of one price. There is also an extensive literature on optimal monetary policy in open economies with nominal rigidities. Some recent contributions include Obstfeld and Rogoff (2000), Devereux and Engel (2003), Corsetti and Pesenti (2005), and the references therein.

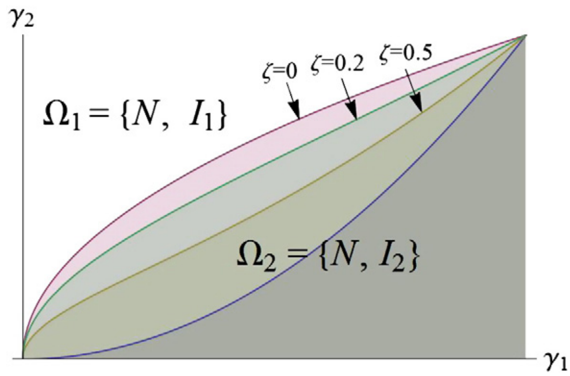

Fig. 5. Equilibrium selection: $\zeta \in [0,0.5)$.

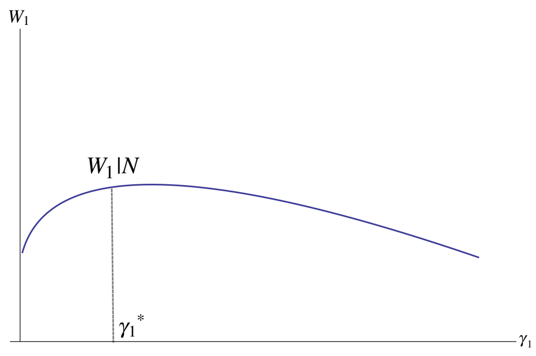

Fig. 7. Case 1: welfare in country 1.

5.2.1. Monetary authorities' choice of (γ_1^*, γ_2^*)

In the first stage of the policy game, each monetary authority selects a money growth rate to maximize welfare for its country, anticipating that citizens respond optimally with $\Theta^*(\gamma_1, \gamma_2)$.

First, I show that monetary authorities can increase welfare by deviating from the Friedman rule, which is the typical optimal policy in single-currency economies without entry externalities. Next I establish existence of subgame perfect money growth rates for a given equilibrium selection mechanism that places some continuity on agents' beliefs in parameter regions with multiplicity. Finally, I construct numerical examples to illustrate the main tradeoffs at hand.

Proposition 2. *Suppose country j fixes its money growth rate at the Friedman rule, $\gamma_j = \beta$. Country $i \neq j$ can benefit by setting its money growth rate above the Friedman rule, $\gamma_i > \beta$, so long as the economy is open ($\alpha < 1$) and there is foreign demand for currency i. That is, $\frac{d\mathcal{W}_i}{d\gamma_i}\big|_{\gamma_i = \beta} > 0$.*

In an open economy, both countries have an incentive to inflate above the Friedman rule if the other country follows the Friedman rule. Since a country can export inflation abroad when foreigners hold its currency, seigniorage becomes a motive for money issue. This temptation to inflate is all the more striking since the monetary authority can resort to lump-sum taxes and does not have any expenditures of its own.

Further, the Friedman rule will only be chosen if $\mathcal{T}_i = 0$. This would be the case in a closed-economy with $\alpha = 1$ and all sellers only accept domestic currency. Since there is only demand for local currency, buyers no longer hold foreign money and hence neither country receives seigniorage payments from the other country. This results in $\mathcal{T}_i = 0$ and $\frac{d\mathcal{W}_i}{d\gamma_i}\big|_{\gamma_i = \beta} = 0$. In that case, inflating will just reduce the purchasing power of currency for its residents, which is the typical distortion in single-currency economies.

Proposition 2 implies that when monetary authorities cannot cooperate and governments always accept domestic currency, the Friedman rule is not the optimal policy. However, determining equilibrium money growth rates requires examining the best response of one country's money growth rate to any arbitrarily given growth rate of the other country, not just the best response to the Friedman rule. When the other country does not follow the Friedman rule, the policymaker must trade off the positive effect of inflation with the negative effect that inflation has on reducing the purchasing power of currency.

Moreover, establishing the existence of subgame perfect money growth rates requires taking a stand on which equilibrium the economy converges to in regions where multiple regimes coexist. For that purpose, I introduce an equilibrium selection mechanism that places some continuity on agents' beliefs. I then show existence for the given equilibrium selection mechanism described.

Definition. *Let $\tilde{\gamma} \equiv \zeta\tilde{\gamma}_1 + (1-\zeta)\tilde{\gamma}_2$, where $\zeta \in [0,1]$ is exogenous. The equilibrium selection mechanism, $\mathcal{G}$, says if $\gamma_1 < \tilde{\gamma}$, regime $\Omega_1 = \{N, I_1\}$ prevails, and if $\gamma_1 > \tilde{\gamma}$, then regime $\Omega_2 = \{N, I_2\}$ prevails.*

The mechanism $\mathcal{G}$ simply introduces an arbitrary rule, $\tilde{\gamma}$, that is a weighted average of the two curves in Fig. 4, $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$. Figs. 5 and 6 plot the rule $\tilde{\gamma}$ for different values of ζ. For example, one specification is that agents believe foreign currency is never accepted for all regions of the parameter space, in which case $\Omega_1 = \{N\}$ and $\Omega_2 = \{N\}$. Another specification is that agents believe currency 1 is international for all $\gamma_1 < \tilde{\gamma} = \tilde{\gamma}_1$ while they believe foreign currency is never accepted when $\gamma_1 > \tilde{\gamma}_1$, in which case $\zeta = 1$, $\Omega_1 = \{I_1\}$, and $\Omega_2 = \{N\}$. The purpose of introducing this mechanism is to place some discipline and monotonicity on agents' beliefs and is not meant to provide ad-hoc microfoundations for how beliefs are actually formed.

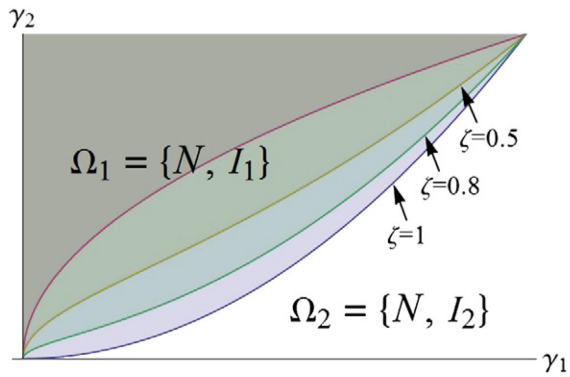

Fig. 6. Equilibrium selection: $\zeta \in (0.5,1]$.

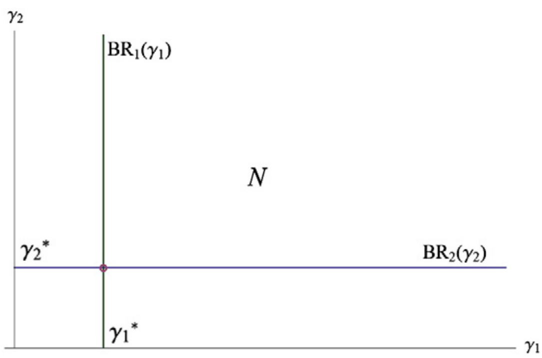

Fig. 8. Case 1: choice of money growth rates.

Proposition 3. *Given $\mathcal{G}$, there exists subgame perfect Nash equilibria and for $\alpha < 1$, it is such that $\gamma_1^* > \beta$ and $\gamma_2^* > \beta$. For $\alpha = 1$, optimal policy is the Friedman rule, $\gamma_1^* = \beta$ and $\gamma_2^* = \beta$.*

I now turn to numerical examples to illustrate the motives of monetary authorities under different scenarios in the last subgame.

Case 1. Foreign currency is never accepted

Consider first the case where agents reject payment in foreign currency for all values of (γ_1,γ_2). That is, $\Omega_1 = \{N\}$ and $\Omega_2 = \{N\}$. Since agents always adopt the trading strategy $(\sigma_1,\sigma_2) = (0,0)$, the welfare function in each country is well-behaved, continuous, and concave. As an example, Fig. 7 plots welfare in country 1 as a function of its policy instrument γ_1, for a given γ_2. In what follows, it is implicitly understood that all endogenous variables are indexed with the regime under consideration. Given country i's welfare when foreign currency is never accepted, monetary authorities choose money growth rates, (γ_1,γ_2),that solves the two country's first-order conditions, $\frac{\partial \mathcal{W}_1|N}{\partial \gamma_1} = 0$ and $\frac{\partial \mathcal{W}_2|N}{\partial \gamma_2} = 0$.

Country 1's best response function is obtained by solving $\frac{\partial \mathcal{W}_1|N}{\partial \gamma_1} = 0$ for their policy instrument, γ_1:

$$\gamma_1 = BR_1(\gamma_1),$$

where $BR_1(\gamma_1) \equiv 1 - \frac{\hat{z}_1}{p'(\hat{q}_1)}$ implicitly depends on its own money growth rate, γ_1, but is independent of country 2's policy instrument, γ_2. This is because when only local currency is accepted, the amount of output traded in one country is determined independently of the amount traded in the other country. Similarly, country 2's best response function is

$$\gamma_2 = BR_2(\gamma_2),$$

where $BR_2(\gamma_2) \equiv 1 - \frac{z_2}{p'(q_2)}$ is also independent of γ_1.

Fig. 8 depicts the two country's best response functions when only local currencies are accepted. In this case, monetary policies are independent and there is a dichotomy between the two currencies. The intersection of $BR_1(\gamma_1)$ and $BR_2(\gamma_2)$ at (γ_1^*,γ_2^*) gives equilibrium money growth rates, which are both strictly above the Friedman rule, (β,β). Since buyers hold both home and foreign currencies (due to their precautionary demand for the latter), the policymaker in each country inflates in order to extract seigniorage from foreigners. This generates an externality for the other country that neither policymaker takes into account. In equilibrium, monetary authorities trade off the gain from inflating with the cost of distorting allocations for its citizens to set an optimal money growth rate that can deviate from the Friedman rule.

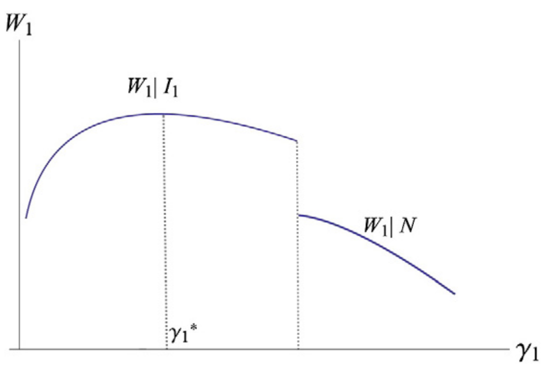

Fig. 9. Case 2: welfare in country 1.

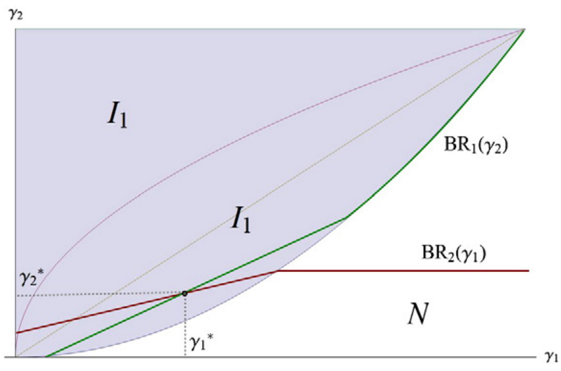

Fig. 10. Case 2: choice of money growth rates.

Case 2. Currency 1 is international

Next consider the equilibrium where sellers from country 2 accept currency 1 when $\gamma_1 < \tilde{\gamma}_1$ and never accept foreign currency otherwise. That is, $\zeta = 1$, $\Omega_1 = \{I_1\}$ and $\Omega_2 = \{N\}$. In this case, both regimes N and I_1 are possible. Although each country's welfare function is continuous within a regime, it is discontinuous at the transition from one regime to another. This is illustrated in Fig. 9, which plots country 1's welfare as a function of its policy instrument, γ_1, for a given γ_2.

Consider the choice of policies when regime I_1 exists. In that case, currency 1 is the sole international currency and within Regime I_1, policymakers' best response functions are obtained by solving the first-order conditions $\frac{\partial \mathcal{W}_1|I_1}{\partial \gamma_1} = 0$ and $\frac{\partial \mathcal{W}_2|I_1}{\partial \gamma_2} = 0$, subject to the constraint that $\gamma_1 \in (\beta, \tilde{\gamma}_1)$. Country 1's best-response function is

$$\gamma_1 = BR_1(\gamma_1, \gamma_2),$$

where $BR_1(\gamma_1,\gamma_2) \equiv 1 - \frac{\hat{z}_1}{z'(\hat{p}_1)} + \frac{(\gamma_2 - 1)p'(q_2) + z_2}{np'(\hat{q}_1)}$ depends on its own policy instrument γ_1 implicitly as well as the other country's, γ_2. Consequently, there is no longer a dichotomy between monetary policies when foreigners accept currency 1 for trade. Similarly, country 2's best-response function is

$$\gamma_2 = BR_2(\gamma_1, \gamma_2),$$

where $BR_2(\gamma_1,\gamma_2) \equiv 1 - \frac{z_2}{p'(q_2)} + \frac{n(\gamma_1 - 1)p'(\hat{q}_1) + \hat{z}_1}{p'(q_2)}$ depends on both γ_1 and γ_2.

Fig. 10 shows an example of the choice of inflation when currency 1 is internationally accepted in region I_1 and locally accepted otherwise, assuming $\alpha = 0.8$. Country 1's best response function is given by $BR_1(\gamma_1,\gamma_2)$ and is always in region I_1 since country 1 has strictly higher welfare by having an international currency than by having local currencies. Moreover, $BR_1(\gamma_1,\gamma_2)$ is increasing in γ_2 since a higher γ_2 implies a higher demand for currency 1, which increases country 1's seigniorage revenue and hence incentive to inflate. Foreign demand leads to seigniorage from abroad, which becomes a motive for money issue. Fig. 9 plots country 1's welfare as a function of γ_1 given γ_2^* and shows a Laffer Curve effect: as inflation rises beyond γ_1^*, the quantity of money demanded falls and the tax base reduced. As a result, there will be an interior money growth rate that maximizes the gain from inflating with the cost of distorting allocations.

Country 2 on the other hand, does not have its currency accepted abroad but also inflates. When γ_2 is in Regime N, its best response function is horizontal since monetary policies are independent when only local currencies are accepted. In Regime I_1 however, $BR_2(\gamma_1,\gamma_2)$ depends positively on γ_1 for the same reason as described above. Equilibrium money growth rates are at the intersection of the two country's best response functions at (γ_1^*,γ_2^*), which are both above the Friedman rule, and lie within the shaded region where currency 1 is international.

5.3. Cooperative policy

I now consider the case where the two monetary authorities cooperate by jointly choosing (γ_1,γ_2) to maximize total welfare for the world. Joint welfare is measured as the sum of the two countries' welfare functions: $\mathcal{W} = \mathcal{W}_1 + \mathcal{W}_2$. Proposition 4 shows that the actions of monetary authorities under cooperation lead to the same equilibrium allocation as the social planner's problem.

Proposition 4. *When monetary authorities cooperate at jointly choosing (γ_1,γ_2) to maximize total welfare for the world, $\mathcal{W}$, the unique optimal policy is the Friedman rule, $\gamma_1^* = \gamma_2^* = \beta$, in which case citizens trade the first-best level of output, q^*, in all matches. As a result, agents never accept foreign currency and hold perfectly diversified portfolios of the two currencies. Equilibrium is socially efficient since no resources are spent on information costs.*

There can be gains from cooperating that are not realized when each country is pursuing its own best interest. When policymakers can coordinate, there are no longer gains from redistributive policies and hence no more temptation to inflate. In this case, the unique equilibrium is the Friedman rule for both countries. Consequently, private citizens only accept their local currency and society saves on information costs.

6. Quantitative analysis

The preceding sections presented a simple two-country, two-currency search model that is amenable to policy analysis. To quantify the welfare cost of losing international status (or the gain of achieving it), the framework is generalized to an arbitrary number of countries and currencies and calibrated to match international trade data. Since much of the set-up and analysis carries over from the baseline model, the N-country, N-currency model is in Appendix A. The model is then used to calculate the welfare benefits of having an international currency for both the issuing country and the rest of the world.

6.1. Calibration

To calibrate the model, the global economy is split into three trading blocs, or regions: the United States, the Eurozone, and China. After discussing parameters that can be easily estimated or fixed independently to their empirical counterparts, I describe the calibration procedure for the remaining parameters. This procedure uses the model's equations and the parameters calibrated independently in order to find parameter values that match moments in the data. All data used are in annual terms from 1999 to 2005 unless otherwise specified.

Following Lagos and Wright (2005), functional forms for utility and cost functions are $U(x) = Bln(x)$, $u(q) = ln(q + b) - ln(b)$, and $c(q) = q$. The parameter b is set to $b = 0.0001$ which ensures a solution to the bargaining problem. The discount factor is set to $\beta = 0.966$, consistent with an annual real interest rate of 3.5 %. Since the model implies that gross money growth rates are also gross inflation rates in a stationary equilibrium, γ_{us}, γ_{eu}, and γ_{ch} are set to average annual inflation rates for the period 1999 to 2005, which is about 2.92 % for the U.S., 1.97 % for the Eurozone, and 5.04 % for China, using data from the World Bank. Different inflation scenarios are also considered in the quantitative exercise.

The bargaining power parameter is set to $\theta = 0.5$ for all regions, consistent with an egalitarian bargaining rule, though I consider the sensitivity of results for different values of θ in Appendix C. Notice that when buyers have all the bargaining power ($\theta = 1$), sellers have no incentive to accept foreign money since they get zero surplus in trade. In that case, there will only be a national currency equilibrium and no welfare gains of having an international currency. As the buyer's bargaining power decreases from unity, it becomes possible for an international money to exist, in which case the welfare gains of having an international currency start becoming positive.

The government size parameters are set to match the fraction of state-owned enterprises in a particular country. The share of state-owned enterprises averaged to less than 5% in the U.S., 22% in Europe, and 37% in China (Szamosszegi and Kyle (2011)). The utility parameter B and relative country sizes are jointly calibrated to match the ratio of each country's GDP over world GDP from Source OECD, which results in $B = 2.03$, $n_{us} = 0.36$, $n_{eu} = 0.37$, and $n_{ch} = 0.27$.

The next set of parameters are the model's meeting parameters for each country pair, μ_{is}. The six international meeting parameters μ_{is} are calibrated with bilateral trade data for the period 1999 to 2005 from the European Commission Bilateral Affairs. Due to an accounting constraint that the total measure of meetings between agents from country i with agents from country s has to be the same as the total measure of meetings between agents from country s with agents from country i, three of the meeting probabilities will not precisely match its targeted value. These values are then backed out using calibrated values for n_i, subject to the accounting constraint.

The final set of parameters is the costs of recognizing foreign currency, which are key for the theory and welfare estimates. To discipline values for information costs, I use data on the extensive margin of foreign currency holdings—whether or not a country holds a particular foreign currency—and how much of a country's trade is denominated in a particular currency. Information on the extensive margin corresponds to private sellers' acceptance decision $\sigma_s = (\sigma_s^{us}, a_s^{eu}, \sigma_s^{ch}) \in \{0,1\}^3$, while international trade invoicing data will partially determine a country's trade composition in different currencies. This approach is consistent with empirical evidence from Friberg and Wilander (2008) that the currency used in trade invoicing is also the one used in actual payment.

In the U.S., only dollars circulate, so that $\sigma_{us} = (1,0,0)$. In the Eurozone and China, dollars are used in international trade invoicing, as reported in Goldberg and Tille (2008). In addition, the Bank for International Settlements reports that U.S. dollars represent most of China's settlement of international trade while the use of euros in China comprises a much smaller share. This results in $\sigma_{eu} = (1,1,0)$ and $\sigma_{ch} = (1,1,1)$.

Next I use data on international trade invoicing to pin down the costs of accepting dollars in Europe and China. Goldberg and Tille (2008) report that the share of dollar-denominated trade in Europe for 2002 ranges from 20.5% in Italy to 71.0% in Greece. I use the reported European average of 32.4% of dollar-invoiced trade to generate the fraction of trades using dollars in the Eurozone. Similarly in Asia, estimates of dollar-denominated trade range from 52.4% to 84.9%. I use the lower bound of 52.4% to determine the fraction of trades using dollars in China. Friberg and Wilander (2008) report an Asia-wide average of 8% of trade denominated in euros.

6.2. Parameter estimates

Tables 4 and 5 summarize the baseline calibration results for the three region model. With this calibration, the model can endogenously generate currency portfolios and quantities traded for a given circulation pattern. With the parameter values from Tables 4 and 5, the model yields two possible types of payment patterns: (i) national currency circulation, and (ii) one international currency (the dollar). As the U.S. inflation increases however, the euro may also emerge as an

Table 4
Parameter values for 3-region model: U.S., Eurozone, and China.

	Interpretation	Value	Target
b	–	0.0001	Ensures solution to bargaining problem
B	Output in CM	2.03	World GDP
β	Discount factor	0.966	Annual real interest rate = 3.5%
γ_{us}	Gross inflation (U.S.)	1.03	Average inflation rate (U.S.) = 3%
γ_{eu}	Gross inflation (Eurozone)	1.02	Average inflation rate (Eurozone) = 2%
γ_{ch}	Gross inflation (China)	1.05	Average inflation rate (China) = 5%
θ	Bargaining power	0.5	Egalitarian bargaining solution
g_{us}	Government size (U.S.)	0.05	Fraction state-owned enterprises (U.S.) = 0.05
g_{eu}	Government size (Eurozone)	0.22	Fraction state-owned enterprises (Eurozone) = 0.22
g_{ch}	Government size (China)	0.37	Fraction state-owned enterprises (China) = 0.37
$\psi_{eu}^{\$}$	Cost of accepting dollars in E.U.	0.024	E.U. trades invoiced in dollars = 32%
$\psi_{ch}^{\$}$	Cost of accepting dollars in China	0.011	China trades invoiced in dollars = 52%
ψ_{ch}^{e}	Cost of accepting euros in China	0.079	China trades invoiced in euros = 8%

international currency alongside the dollar, as discussed in more detail in Appendix C.

7. Welfare benefits of international currency

There are normative consequences of a switch from one type of monetary regime to another, as first discussed in Section 4.5, and these changes will generate real gains and losses for all countries in terms of economic welfare. To study the welfare effects of potential shifts in payment patterns, I follow the approach of Lucas (1987) and ask how much consumption citizens demand or are willing to give up as compensation to move from regime $\Omega \in \{N, I_{us}, I_{eu}\}$ to another regime $\Omega' \neq \Omega$, where regime N denotes an equilibrium with only national currencies, regime I_{us} is an equilibrium where only the dollar is international, and regime I_{eu} is an equilibrium where only the euro is international.

Under a given regime Ω, steady-state welfare in each region $i,j,\ell \in \{us, eu, ch\}$ is measured as the steady-state sum of buyers' and sellers' surpluses, weighted by their respective sizes in the DM. Welfare for region i in regime Ω is

$$\begin{aligned}\mathcal{W}_i(\Omega) = {} & U(x^*\Omega) - x^* + \mathcal{B}_i\mathcal{T}_i + \alpha_{ii}\mathcal{M}_i\left\{(1-g_i)\sum_k \sigma_i^k S\left(q_{ii}^k(\Omega)\right) + g_i S(Q_i i(\Omega))\right\} \\ & + \alpha_{ij}\mathcal{M}_j\theta\left\{\left(1-g_j\right)\sum_k \sigma_j^k S\left(q_{ij}^k(\Omega)\right) + g_j S\left(Q_{ij}(\Omega)\right)\right\} \\ & + \alpha_{ji}\mathcal{M}_i(1-\theta)\left\{(1-g_i)\sum_k \sigma_i^k S\left(q_{ji}^k(\Omega)\right) + g_i S\left(Q_{ji}(\Omega)\right)\right\} \\ & + \alpha_{i\ell}\mathcal{M}_\ell\theta\left\{(1-g_\ell)\sum_k \sigma_\ell^k S\left(q_{i\ell}^k(\Omega)\right) + g_\ell S(Q_{i\ell}(\Omega))\right\} \\ & + \alpha_{\ell i}\mathcal{M}_i(1-\theta)\left\{(1-g_i)\sum_k \sigma_i^k S\left(q_{\ell i}^k(\Omega)\right) + g_i S(Q_{\ell i}(\Omega))\right\} \\ & -(1-g_i)\mathcal{S}_i\sum_k \sigma_i^k \psi_i^k,\end{aligned}$$

where $S(q^k(\Omega)) \equiv u(q^k(\Omega)) - c(q^k(\Omega))$ is the total trade surplus given that private sellers recognize currencies k in regime Ω. A derivation of this welfare function is in Appendix A. Welfare depends on three components: (i) net consumption in the CM, (ii) net seigniorage revenue given by $\mathcal{B}_i\mathcal{T}_i$, and (iii) surplus from trading in the decentralized markets net of any information costs.

Suppose the economy moves from Ω to a different equilibrium Ω', but also adjusts consumption of all goods in the CM and DM by a common factor, Δ_i. The amount $1-\Delta_i$ then measures the percentage gain, or loss if $1-\Delta_i < 0$, of consumption faced by agents in country i per year. The compensating variation value $1-\Delta_i$ that solves $W_i(\Omega') = W_i(\Omega)$ is then the welfare benefit or cost of moving from regime Ω to Ω'.

Table 6 summarizes the annual consumption equivalent welfare changes for transitions across various steady-state equilibria. For the U.S., the welfare benefit of having the dollar as the sole international currency ranges from 0.7% to 1% of consumption per year depending on the U.S. inflation rates. This gain derives from two sources. The first source is from increased seigniorage revenues abroad. In practice, foreigners hold much of the U.S. currency stock: Porter and Judson (1996) report that approximately 60% of dollar banknotes are held abroad. In turn, the model implies that the flow of international seigniorage to the U.S. is approximately 0.1–0.2% of GDP, in line with estimates from Portes and Rey (2002) and Goldberg (2011). The second source however comes from the model's general equilibrium effects of increased international exchange: due to increased acceptability of the dollar, there is now more surplus from international transactions. As more people use the dollar, its value goes up, which increases the amount of goods that can be purchased for a given unit.

Table 6 also highlights the distributional effects of inflation across countries. For the issuing country, some inflation can be beneficial due to increased seigniorage. However this harms foreigners who have to incur a higher inflation tax. Monetary policy can therefore have distributional effects across countries by redistributing wealth from foreigners to domestic agents.

In Appendix C, I consider an alternative calibration procedure for parameterizing the model's information costs that use data on the shares of dollars and euros in outstanding international debt securities (World Bank (2011), ECB (2011)). This data is used to discipline values for the model's information costs so that the model-implied shares of dollars and euros match the data.

Under this alternative calibration, total welfare gains to the U.S. of having the dollar as the sole international currency are lower, ranging from 0.4 to 0.7% of GDP per year. The main reason is due to the relatively

Table 5
Calibrated meeting parameters: U.S., Eurozone, and China.

Parameter	Estimate	Target moment	Model	Data
n_{us}	0.36	GDP share (U.S.)	19.3%	19.1%
n_{eu}	0.37	GDP share (Eurozone)	19.7%	19.8%
n_{ch}	0.27	GDP share (China)	13.9%	14.3%
$\mu_{us,eu}$	0.238	Share of U.S. trade with E.U.	11.7%	17.8%
$\mu_{us,ch}$	0.108	Share of U.S. trade with China	10.3%	14.9%
$\mu_{eu,us}$	0.186	Share of E.U. trade with U.S.	14.0%	13.8%
$\mu_{eu,ch}$	0.241	Share of E.U. trade with China	15.4%	13.3%
$\mu_{ch,us}$	0.227	Share of China trade with U.S.	13.8%	13.6%
$\mu_{ch,eu}$	0.268	Share of China trade with E.U.	17.0%	17.0%

Table 6
Welfare changes in consumption equivalent terms (% of GDP per year).

	$\gamma_{US} = 1.03$	$\gamma_{US} = 1.05$	$\gamma_{US} = 1.06$	$\gamma_{US} = 1.10$
	N to I_{US}	N to I_{US}	I_{US} to I_{EU}	I_{US} to I_{EU}
$1-\Delta_{US}$	0.73	1.02	−0.93	−1.10
$1-\Delta_{EU}$	0.21	0.18	1.22	1.36
$1-\Delta_{CH}$	0.22	0.19	0.24	0.20

larger role played by the euro in denominating global debt versus its role in trade invoicing. Less residents holding dollars and more holding euros produces a larger (smaller) calibrated cost of accepting dollars (euros) than under the baseline calibration. This translates to a direct reduction in welfare for the fraction of sellers who invest to accept dollars and moreover, an indirect reduction in welfare for buyers in the rest of the world as now less sellers accept dollars and hence the benefit from holding dollars is also less.

That welfare estimates are sensitive to the quantities used to calibrate the model's information costs is an interesting finding since these parameters are likely to have decreased over time and may well be expected to decrease further in the future. The quantitative analysis illustrates that a reduction in information cost can lead to a welfare improvement for the issuing country that may even be higher than the welfare gain from seigniorage revenue (typically, 0.1–0.2% of annual consumption). As well, the rest of the world may also benefit both directly through lower information costs and indirectly through more trading opportunities.

While the exact quantitative results presented here may be sensitive to the assumed pricing mechanism or calibration strategy, the model illustrates how having microfoundations for international payments can have quantitatively important implications for welfare. For brevity however, further investigation of the model's quantitative implications is delegated to future research.

8. Conclusion

This paper provides an information-based theory of international currency by generalizing the model of asset liquidity by Lester et al. (2012) to an open-economy setting. I investigate some classic issues in international monetary economics, such as the emergence of an international currency, the choice of inflation when currency acceptability is endogenous, and the welfare benefits of international currency use. Instead of assuming the payments used in each country, the model allows private citizens to choose which currencies to accept. Further, government transaction policies are introduced to examine how certain policies—namely ones which favor the use of a country's national money—affect private agents' acceptance decisions and hence the set of equilibria. Fairly innocuous policies of the kind considered ended up implying the connections observed in practice between currencies and countries.

This paper also explicitly modeled the strategic interaction among money issuers in a dynamic policy game. An inflation Laffer curve emerges and captures the main tradeoffs faced by the issuing country. While some inflation can be welfare-improving by increasing the amount of seigniorage extracted from foreigners, too much inflation lowers the purchasing power of money. Since sovereign policymakers are only responsible for welfare of their own citizens, they are not penalized for any negative effects that their policies may have abroad. If however externalities can be internalized in a cooperative agreement, then all countries may benefit.

Quantitatively, the welfare cost of losing international status is not inconsequential for the issuing country. For the U.S., this amounts to about 0.4% to 1.1% of consumption each year, depending on the calibration strategy. This paper thus provides a first step in examining the effects of transitioning across different types of payment regimes using a microfounded model where credit is imperfect and accepting foreign currency is costly.

Consistent with evidence from Eichengreen (2010), this paper questions the conventional wisdom that competition for international currency status is a winner-take-all game. Just as history shows that several international currencies have often shared this role in the past, the theory implies that a likely situation for the future monetary system is one where several international currencies compete and coexist.

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.jinteco.2014.04.005.

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