

# MORTGAGE PREPAYMENT FLOAT: PRICING AND RISK ANALYSIS

LAZARUS A. ANGBAZO, JOHN J. McCONNELL,  
ISAAC F. MEGBOLUGBE, AND TYLER T. YANG

**T**his article is a theoretical and empirical analysis of the value of mortgage servicers' net prepayment float. Net prepayment float is the investment income on prepaid principal and interest prior to remittance to the ultimate mortgage note holder. Float income is potentially an important component of servicers' overall profitability, but its value and risk cannot be estimated easily. We develop a pricing model of float and use it to empirically analyze the value and risk of float during a thirteen-year sample period.<sup>1</sup>

For a fee, a mortgage servicer performs the role of managing the mortgage payment process for an investor or the ultimate purchaser of the mortgage note. Servicing includes collection of monthly payments from borrowers, the transfer of principal and interest payments to investors, the management of escrow accounts, and the handling of delinquencies and foreclosures.<sup>2</sup>

The servicing fee varies with the type of loan, but ranges between 25 and 50 basis points of the unpaid balance. Occasionally, servicers earn additional income from late fee charges assessed against delinquent borrowers. In addition, servicing provides a valuable customer base for loan and deposit products, insurance, and investment services that have the potential for materially influencing the returns and risks of mortgage lenders. In short, the value of the servicing contract is a significant component of the overall profitability of primary mortgage lenders. One aspect of that profit is the value of income received from the mortgage flow.

The value and volatility of float income depends upon prepayments of the loan being serviced (see Rosenblatt [1994]). On the positive side, a servicer earns significant prepayment float income by investing

**LAZARUS A. ANGBAZO** is vice president of portfolio management at Chase Manhattan Bank in New York.

**JOHN J. McCONNELL** is a professor of finance at Purdue University.

**ISAAC F. MEGBOLUGBE** is with the Fannie Mae Foundation.

**TYLER T. YANG** is with Price Waterhouse.

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funds between the time borrowers prepay the remaining balance of a loan and the servicer remits the funds to the ultimate holder of the mortgage note. On the negative side, prepayments accelerate interest losses on MBS pools, which represent the amount of uncollected interest that servicers may be required to pass on to the securityholder when a loan is not paid off on the last day of the month.

Thus, prepayments provide both a benefit and a cost. Both effects depend on the remittance pattern required by the ultimate investor, the level and volatility of interest rates, the underlying mortgage pass-through coupon rate, and the expected conditional prepayment rates of the mortgage.

The purpose of this article is twofold. First, we use a contingent claim framework to value the net prepayment float income that is embedded in the standard servicing contract associated with Fannie Mae and Freddie Mac loan pools — the two dominant customers of servicing contracts in the market. The valuation model uses the fact that the cash flows from the prepayment float programs are analogous to those of a discount bond. The value of float is determined by the term structure of interest rates and the within-month distribution of prepayments when they occur. The Cox-Ingersoll-Ross [1985] term-structure model is invoked to characterize the underlying interest rate process, and a prepayment function is empirically estimated from recent data. The valuation model provides a useful analytical benchmark for valuing the float embedded in specific servicing contracts and for estimating risks of the float.

Second, we use an explicit finite difference method as proposed by Hull and White [1990] to obtain numerical solutions to the prepayment float valuation equation. We use the trinomial lattice procedure to estimate the value of float for the standard Fannie Mae and Freddie Mac contracts for thirty-year fixed-rate mortgages during the 1983-1995 period. Historical interest rate parameters allow us to compute the theoretical value of float. We also vary these parameters around the historical levels to infer the relationship of the value and risk of float to changes in market interest rates.

In general, the results show that Freddie Mac's float program, which insures servicers against prepayment interest obligations, is more favorable when prepayment risk is high. Specifically, the float value is greater when short-term market rates and the mean reversion level are low, and when interest rate volatility

and mortgage pass-through rates are higher. By comparison, servicers are better off with Fannie Mae's servicing contracts when short-term rates and the mean-reverting level are high, and when the market interest rate volatility is low. Under such conditions, the longer float benefit period outweighs the interest obligations from prepayments.

First we review the prepayment remittance programs of Fannie Mae and Freddie Mac, highlighting the potential float benefit and the amount of prepayment rate risk of the servicers. We then develop an interest rate-contingent claim pricing model for the prepayment float based on the Cox-Ingersoll-Ross mean-reverting square root process. We provide empirical and simulation results of the relationship of the net prepayment float to market interest rate conditions for the mortgage purchase commitment offers of Fannie Mae and Freddie Mac. Our conclusions have some implications for servicers' mortgage contracts.

## I. PREPAYMENT FLOAT REMITTANCE PROGRAMS

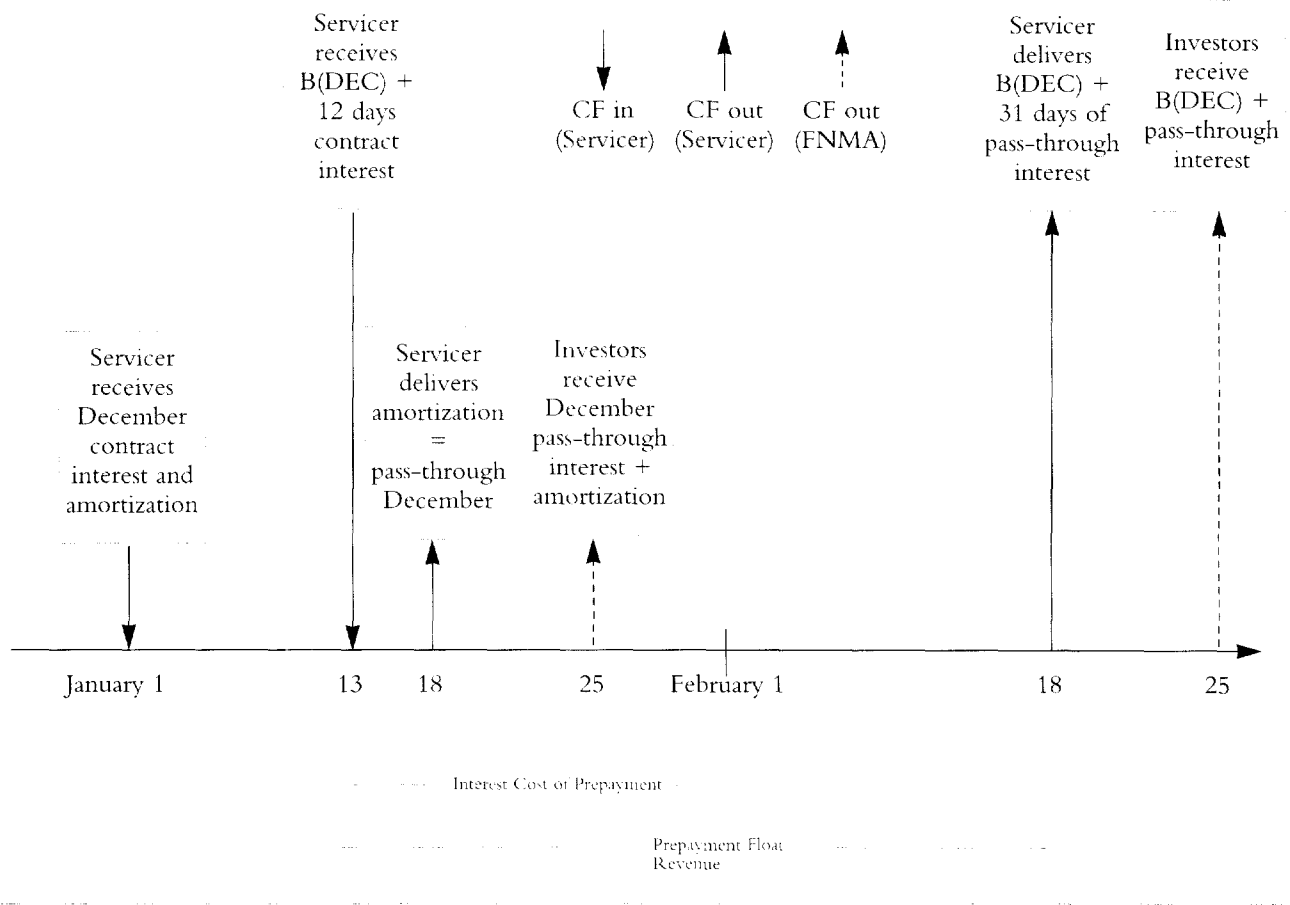
The loan purchase commitments in which secondary mortgage market agencies purchase individual mortgage loans from originators typically specify the MBS price and the required default guarantee fee in terms of basis points. If servicing rights are retained by the originator, the commitment also specifies the period for which servicers earn market interest on payoffs received and the interest obligations due if prepayments occur. This period is usually referred to as the float period.

The remittance schedules of Fannie Mae and Freddie Mac provide different float periods. Similarly, in case of prepayments, the servicing contracts of the two institutions expose servicers to different levels of interest rate risk.

### The Fannie Mae Program

Exhibit 1 provides a graphical illustration of the standard Fannie Mae remittance schedule. The standard program requires the servicers to remit prepaid principal and the accrued pass-through interest on the eighteenth day of the month following the prepayment month. Exhibit 1 assumes that a borrower prepays the mortgage on January 13, where  $B(\text{Dec})$  represents the unpaid balance as of December 31. The servicer receives an amount  $B(\text{Dec})$  plus twelve days of accrued

**EXHIBIT 1 ■ Mortgage Payoff Remittance Schedule for Fannie Mae**



interest at the contract rate on January 13. Fannie Mae requires the servicer to deliver the prepaid principal plus the entire scheduled mortgage payment for that month, however, even though prepayment has occurred. Thus, the servicer delivers the amount  $B(\text{Dec})$  plus thirty-one days of accrued interest at the pass-through rate on February 18.

An implication of the remittance program is that the servicer must “carry” the mortgage for up to one month at the interest rate of the prepaid mortgage. Since the mortgage pass-through rate is typically higher than the short-term interest rate, the servicer is exposed to an interest income shortfall.

There is, however, a compensating feature of the contract that potentially offsets the interest income shortfall. Under Fannie Mae’s program, servicers are allowed to retain the prepaid principal and interest until the eighteenth of the following month. In theory, servicers can earn interest income at the prevailing market

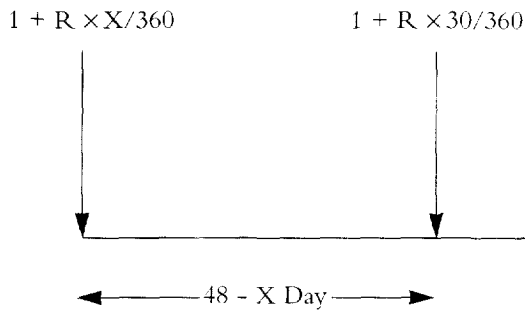
rate for up to forty-eight days. The net impact of the remittance program depends on which of the two offsetting factors is larger.

In Exhibit 1, ignoring regular payment for December and the Fannie Mae remittance to MBS investors, the prepayment float translates into a discount loan of  $B(\text{Dec})[1 + R \times x/360]$  that is payable in  $48 - x$  days at a face value of  $B(\text{Dec})[1 + R \times 30/360]$ , where  $R$  is the annual pass-through coupon rate of the prepaid mortgage, and  $x$  indicates that the prepayment occurs on the  $(x + 1)^{\text{th}}$  day of the month. To keep the discussion simple, we assume a 30-day month and a 360-day year. The corresponding cash flows are described in Panel A of Exhibit 2.

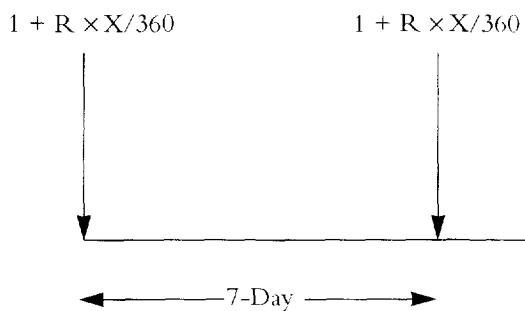
There are two types of risks associated with the float: interest rate risk and timing risk. The interest rate risk is contributed by the volatility of the market interest rates. During the float period, the servicing agent needs to earn enough float revenue to cover the differ-

**EXHIBIT 2 ■ Net Prepayment Float**

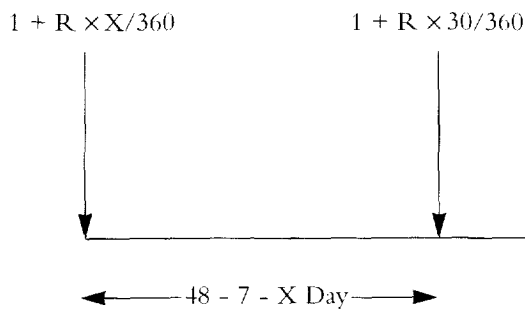
Panel A. Fannie Mae



Panel B. Freddie Mac



Panel C. Fannie Mae versus Freddie Mac



ence between the actual prepayment amount (principal plus coupon interest up to the prepayment date) and the amount to be passed to the MBS holders (principal plus coupon interest for the entire month). The servicing agent may lose money during this float period if the pass-through rate is significantly higher than the market rate at which the servicer may invest.

The timing risk refers to uncertainty about the time a prepayment will occur. If the prepayment occurs early in a month, both the interest shortfall and the float revenue are higher. Under normal conditions, the

incremental increase of costs outweighs the incremental increase of revenues. Thus, loss is more likely if prepayment occurs early in a month, while profits are almost assured if the prepayment occurs close to the end of a month.

**The Freddie Mac Program**

Exhibit 3 depicts the standard Freddie Mac program. Freddie Mac's program requires that the exact amount received by the servicer be delivered to Freddie Mac within five business days (or seven calendar days), whether payoffs are regular payments or prepayments. For tractability, we ignore the day-of-the-week effect, and assume the period is always seven calendar days. In Exhibit 3, if a borrower prepays on January 13, the servicer receives  $B(\text{Dec})$  plus twelve days of coupon interest. The servicer, in turn, remits  $B(\text{Dec})$  plus twelve days of pass-through interest on the twentieth, since the remittance policy guarantees seven days of float at the market rate. A major advantage of Freddie Mac's program is that there is no interest shortfall related to prepayment. Mortgage prepayments represent pure float benefits to the servicers.

Again ignoring regular principal and coupon interest payments of December and the remittance cycle to MBS investors, the prepayment float program is analogous to a discount loan of  $B(\text{Dec})[1 + R \times x/360]$  that is payable in seven days at a face value of  $B(\text{Dec})[1 + R \times x/360]$ . Panel B of Exhibit 2 shows the cash flows associated with prepayment float under the Freddie Mac program.

**Fannie Mae versus Freddie Mac**

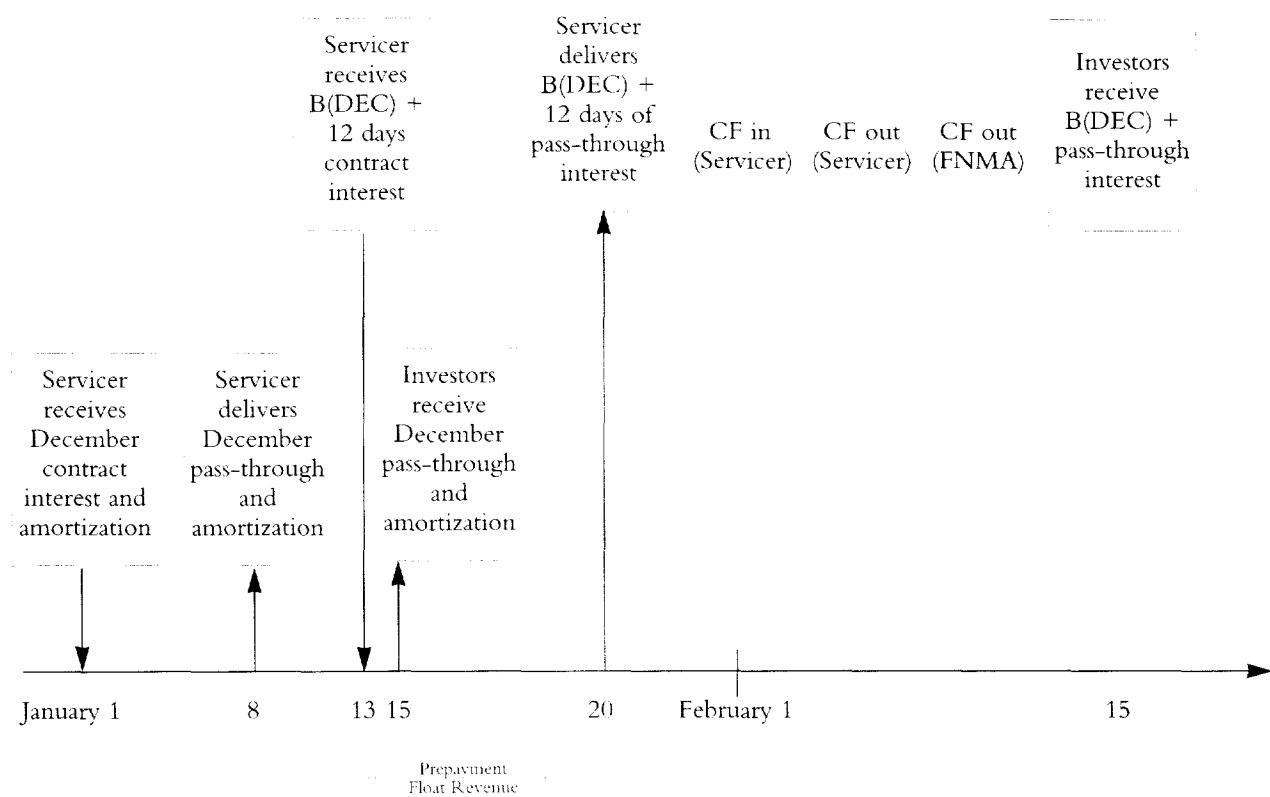
The advantage of the Fannie Mae program over the Freddie Mac program can be seen by taking the difference between the cash flows in Panel A and Panel B of Exhibit 2. The difference, shown in Panel C, is analogous to a discount bond. The servicer receives a loan from Fannie Mae for the amount  $[1 + (x/360)R]$  at the prepayment date and promises to repay the amount  $[1 + (30/360)R]$  in  $48 - 7 - x$  days.

The easiest way to analyze the loan is to compute the yield as:

$$y = \left[ \frac{1 + 30R/360}{1 + xR/360} \right]^{360/(41-x)} - 1 \quad (1)$$

If the yield  $y$  is lower than the average yield of short-term investments during the float period, the float with

**EXHIBIT 3 ■ Mortgage Payoff Remittance Schedule for Freddie Mac**



the Fannie Mae program is more valuable than that with the Freddie Mac program, and vice versa. The yield  $y$  increases with  $R$  and decreases with  $x$ .

Prepayment (refinancing) is more likely to occur when the market rate drops substantially. As the interest rate drops, the coupon rate of the mortgage,  $R$ , and the yield,  $y$ , become high relative to the market short-term rate. This makes float in the Freddie Mac program more profitable than that in the Fannie Mae. When we take this together with the higher prepayment probabilities during such market conditions, there is a perception that the Freddie Mac program provides a greater value. As will become clear in the analysis, however, this perception may be misleading in many interest rate scenarios.

**II. THE VALUATION MODEL**

The float can be valued as an interest rate-contingent claim. The valuation model that we propose requires specification of a model of the term structure of interest rates. For purposes of illustration, we assume

that the short-term interest rate follows the Cox, Ingersoll, and Ross [1985] mean-reverting square root process. Other term structure models, such as Hull and White [1990] and Heath, Jarrow, and Morton [1992], also can be easily used for developing the prepayment float valuation model.<sup>3</sup>

The CIR process has the specification

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz \tag{2}$$

where  $r$  is the instantaneous short-term interest rate,  $\kappa$  is the rate of reversion of  $r$  toward  $\theta$ ,  $\theta$  is the long-term mean-reverting level of  $r$ , and  $\sigma$  is the volatility of  $r$ . CIR show that with this interest rate process, all interest rate-contingent claims must satisfy the partial differential equation

$$\frac{1}{2}\sigma^2r^2P_{rr} + \kappa(\theta - r)P_r + P_t - \lambda rP_r - rP = 0 \tag{3}$$

where  $\lambda$  is market price of risk, and  $\lambda = 0$  implies

risk-neutrality.

The present value of a \$1 face value discount bond at time  $t$ , with market rate  $r$  and payoff date  $s$ , is

$$P(r, t; s) = A(t, s)e^{-rB(t, s)} \quad (4)$$

where

$$A(t, s) = \left[ \frac{2\gamma e^{(\kappa + \lambda + \gamma)(s-t)/2}}{(\kappa + \lambda + \gamma)[e^{\gamma(s-t)} - 1] + 2\gamma} \right]^{\frac{2\gamma\theta}{v^2}}$$

$$B(t, s) = \frac{2[e^{\gamma(s-t)} - 1]}{(\kappa + \lambda + \gamma)[e^{\gamma(s-t)} - 1] + 2\gamma}$$

$$\gamma = \sqrt{(\kappa + \lambda)^2 + 2\sigma^2}$$

For prepayment float, if a prepayment occurs on the  $x^{\text{th}}$  day of the month, the Fannie Mae program has  $t = x/360$  and  $s = (48 - x)/360$ , and the principal amount equals  $1 + R/12$  of the beginning balance. With Freddie Mac,  $t = x/360$ ,  $s = (x + 7)/360$ , and the principal amount equals  $1 + (xR)/360$ . The profit (loss) to the servicer under Fannie Mae is

$$\pi_x = \left(1 + \frac{xR}{360}\right) - \left(1 + \frac{R}{12}\right)P(r, x; \frac{48-x}{360}) \quad (5)$$

where  $\pi_x$  represents the profit of the servicer under the Fannie Mae program.

The profit (loss) to the servicer under the Freddie Mac program is

$$\pi_x = \left(1 + \frac{xR}{360}\right) - \left(1 + \frac{R}{12}\right)P(r, x; \frac{x+7}{360})$$

With prepayments occurring throughout the month, the value of the float at a node can be computed as the weighted average of the net present values realized on different prepayment days  $x$ . That is,

$$F = \sum_{x=1}^{30} w_x \pi_x P(r, 0; x) \quad (6)$$

where  $w_x$  is the percentage of total prepayments that

occur on the  $x$ -th day of the month.

Equation (6) gives the value of the float at a particular month, given that the prepayment does occur. Thus, the payoff function of the prepayment float at a given time is

$$F_t = \begin{cases} F & \text{if prepayment occurs} \\ 0 & \text{otherwise} \end{cases}$$

To implement this model, we use Chen and Yang's [1995] prepayment function, which assumes rational prepayment behavior. Their model is an extension of Navratil's [1985] logistic prepayment function:

$$Z_i = \ln\left(\frac{\psi_i}{1 - \psi_i}\right)$$

where  $Z_i$  is a function of the economic factors  $X_i$ , and  $\psi_i$  is the probability of prepayment under economic condition  $i$ . The  $Z$  function is specified empirically using the actual monthly prepayment rates of GNMA pools between January 1985 and August 1989 as reported in Salomon Brothers' publication, *Mortgage Security Prepayment Rate Profile*. The derived  $Z$  function is therefore

$$Z = -5.05 + 0.57X_1 - 0.33X_2 - 0.59X_3 + 0.15X_4 \quad (7)$$

where  $X_1$  is the 100% Public Securities Association (PSA) experience of the given mortgage age,  $X_2$  is the interest rate spread between the pass-through rate and the short rate plus 5%,  $X_3$  is the dummy variable indicating whether  $X_2$  is positive or negative, and  $X_4$  is an interactive term equal to  $X_2$  times  $X_3$ .

The actual prepayment rate can be calculated as:

$$\psi = \frac{\exp(Z)}{1 + \exp(Z)}$$

This estimated prepayment rate is bounded between zero and one. This function is used in our model to capture the asymmetric distribution of the prepayment rate with respect to market interest rates.

Numerical methods are used to solve for the value of the float. The explicit finite difference method proposed by Hull and White [1990] is applied to construct a trinomial tree of interest rates. The value of the float at time  $t$ , state  $i$ , in the tree is calculated as

$$V_{t,i} = \begin{cases} \Psi_{t,i} F_{t,i} + (1 - \Psi_{t,i}) P(r_i, t; t + \Delta t) \sum_{j=k-1}^{k+1} \Omega_{t,j} F_{t+\Delta t, j} & \text{if } t < T \\ 0 & \text{if } t = T \end{cases} \quad (8)$$

where  $r_i$  is the interest rate at state  $i$ ,  $\Omega_{t,j}$  is the risk-neutral probability of the interest rate moving from state  $i$  to state  $j$  in  $\Delta t$ , and  $T$  is the maturity date of the mortgage.

Note that  $r$  is time-independent because the CIR process is a fixed parameter process. If other time-dependent interest processes are used, then both  $r$  and  $\Omega$  will be time-dependent. Similarly, depending on the mean reversion speed and the mean reversion level of the process, Hull and White have specified five possible branching schemes to guarantee the non-negativity of the risk-neutral probabilities. These five possible branch schemes correspond to the cases where  $k = i - 2, i - 1, i, i + 1$ , and  $i + 2$ . With this recursive equation and the trinomial interest rate tree, the value of the float is estimated by a backward recursive solution technique.

### III. RESULTS

The results are based on simulations comparing the values of the prepayment float embedded in the servicing contracts of Fannie Mae and Freddie Mac. The float valuation model is used to compute the value of the float associated with thirty-year fixed-rate mortgages originated in each month during January 1983 through October 1995.

#### Parameter Estimation and Data

Implementation of the valuation formula requires that we estimate the parameters of the underlying term structure model. First, the base values of model parameters are set to the term structure parameters estimated by Chan et al. [1992] from historical data. These are speed of mean reversion  $\kappa = 0.2339$ , level of mean reversion  $\theta = 0.0808$ , and interest rate volatility  $\sigma = 0.0854$ . The risk preference parameter  $\lambda$  is estimated as a time-varying characteristic to obtain a better fit of the term structure for each month. To compute the implied risk preference, we use the ten-year Treasury yields  $r_t^{10}$  reported by the Department of the Treasury. At each month, we search by iteration for  $\lambda$  for which the ten-year Treasury bond with a coupon rate equal to  $r_t^{10}$  would be priced at par.

The coupon bond price can be written as the sum of various discount bonds:

$$CB(r, t; s) = \sum_{i=t+1}^s c_i P(r, t; i)$$

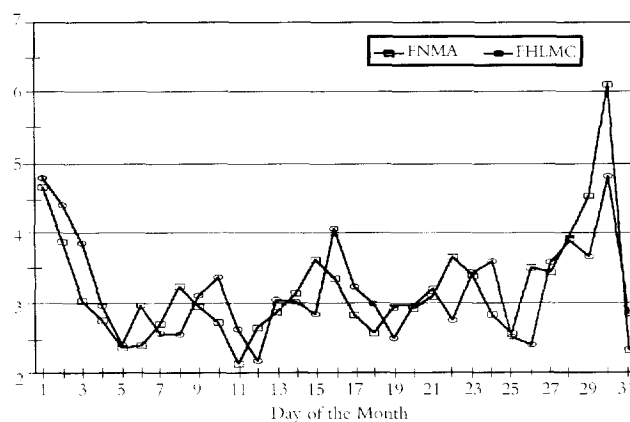
where  $c_i$  is the cash flow for each period (semiannual) until the maturity date  $s$ . During the sample period,  $\lambda$  ranges between  $-0.0308$  and  $0.0066$ , with an average of  $-0.0081$ .

The thirty-year fixed mortgage contract rates are obtained from the Federal Home Finance Board survey data. The three-month Treasury yield reported is the proxy for the market short-term rates  $r_t$ . The prepayment data are the actual prepayment experience of Fannie Mae and Freddie Mac for thirty-year fixed-rate mortgages. The model assumes that prepayments are uniformly distributed across the days in a month. This assumption is based on the empirical prepayment data of Fannie Mae and Freddie Mac thirty-year fixed-rate mortgages. Exhibit 4 shows that the prepayments are evenly distributed during a month.

#### Prepayment Float Value

Since both the volume and the variety of servicing contracts offered by secondary mortgage market institutions have grown over time, the ability to analyze the value of loan servicing contracts has become an important determinant of a seller/servicer's choice among contracts.<sup>4</sup> We use the valuation model to com-

EXHIBIT 4 ■ Distribution of Prepayments in a Month



pute the value of the net prepayment float for the Fannie Mae and Freddie Mac programs under various interest rate and prepayment conditions. The empirical results illustrate how the pricing model can be used to evaluate alternative servicing contracts.

**Historical Patterns.** Exhibit 5 displays the computed monthly value of prepayment float on newly originated mortgage pools from 1983 through October 1995. Because they are based on prevailing interest rates and prepayment experience, the computed values reflect actual float experience. The float valuation mirrors the volatility in market rates during the sample period.

The trend during 1983-1984 shows a prepayment float valuation that is more favorable for Freddie Mac than for Fannie Mae. Although interest rates were relatively high during 1983-1984, the rates reflect a drop of 300 basis points from the previous high rate environment (1979-1982). The higher value of the float from Freddie Mac reflects the fact that its servicing contracts were fully insulated from the impact of falling interest rates. The value of the float embedded in Fannie Mae contracts reflects a drop of over 300 basis points from the 1979-1982 market highs.

As interest rates fell even farther beginning in 1985, followed by the refinancing boom in 1986-1987, float income under both contracts was negatively affected. In the late 1980s and early 1990s, the float value was relatively stable. The float benefits of both contract types stayed within the 3.5 to 4.5-basis point range. Starting from 1992-1993, however, when mortgage rates dropped approximately 400 basis points and prepayments increased, new servicing contracts were valued more favorably under the Fannie Mae program. The average spread in monthly float income during the 1992-1993 period was about 3 basis points; it rose in later years as refinance activity persisted in 1993-1994.

At first blush, the superior value of float of the Fannie Mae program in recent years appears to be paradoxical. Because interest rates fell and refinance activity increased, we might expect the resulting interest obligations to exceed the value of the float revenue earned from prepayments. Yet because the expected float values are computed for new-issue mortgages, while the interest cost of current prepayments is on outstanding mortgages, the expected net prepayment float value for the Fannie Mae program can still be greater if the new-issue mortgages have lower expected prepayment speeds. Recently originated mortgages should have lower prepayments, and hence less of a risk

EXHIBIT 5A ■ Monthly Market Rates

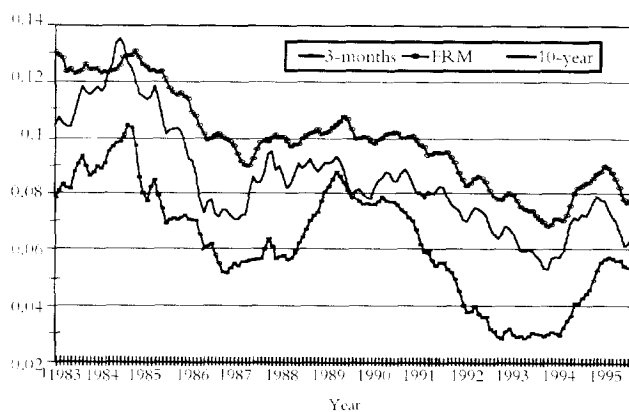
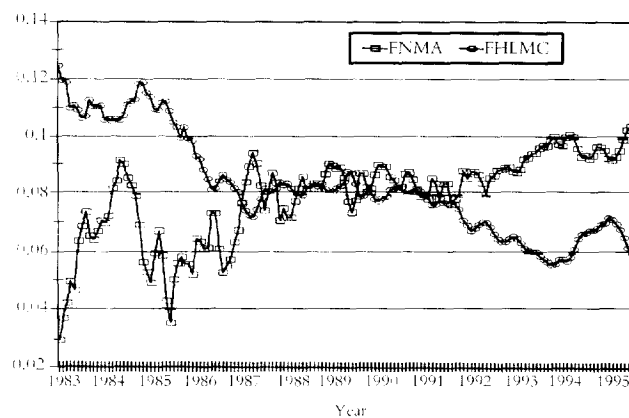


EXHIBIT 5B ■ Historical Monthly Net Prepayment Float



of yielding a negative net prepayment float income, since the mortgages have lower rates.

**Effect of Interest Rates.** Several properties of interest rates are simulated to determine their impact on prepayment float. These include the level of interest rates  $r$ , the long-term mean-reverting level  $\theta$ , and the conditional volatility of interest rates  $\sigma$ . Exhibit 6 compares the expected value of prepayment float income for Fannie Mae and Freddie Mac contracts at different short-term interest rates prevailing at origination. Similarly, the impact of mean reversion is illustrated by the relationship of float to  $\theta$ . The curves marked FNMA(8) and FHLMC(8) represent the expected float values when the mean-reverting level of short-term interest rates is 8% (i.e., below the average historical level).

The expected float benefits for the Fannie Mae program when market rates are low are lower than



those for the Freddie Mac program. The difference reflects the differential prepayment interest cost burden at the low interest rates where Freddie Mac contracts are fully insulated. At higher market rates, this advantage is offset by the decline in prepayments that, in turn, provides for higher float under Fannie Mae contracts.

Exhibit 6 also illustrates the differential impact of the mean reversion level. Specifically, when the mean reversion level is higher than the historical pattern, there is a dampening effect on the net prepayment float of Freddie Mac, as indicated by the lower curve FHLMC(12) compared to FHLMC(8). Fannie Mae contracts provide higher float value at higher-than-historical mean-reverting levels of short-term rates, possibly reflecting the fact that prepayment interest rate risk is minimized at higher market rates.

In Exhibit 7, the values of the term structure parameters  $\lambda$  and  $\sigma$  are allowed to vary around their historical levels to illustrate their likely impact under changing environments. Recall that the historical levels are based on the empirically estimated values of Chan et al. [1992]. The results produce a number of interesting relationships.

First, both firms' float values decrease monotonically as market volatility increases, but Fannie Mae's contracts exhibit greater sensitivity. In the lower-volatility environment (i.e.,  $\sigma$  less than the average historical level), Fannie Mae float is more valuable than Freddie Mac's. This is probably because the longer float period offsets the interest obligation due from prepayments.

Second, the value of Freddie Mac float is relatively invariant to  $\lambda$ , while Fannie Mae's is not. The  $\lambda$

#### EXHIBIT 7 ■ Value of Float Income for Various Values of Volatility ( $\sigma$ ) and Risk Preference ( $\lambda$ )

##### Fannie Mae

Value of Float Income per \$100 MBS Face Value

$\kappa$	$\sigma$	$\lambda$			
		-0.02	-0.01	0	0.005
0.25	0.07	0.0454	0.0405	0.0352	0.0324
	0.09	0.0394	0.0353	0.0309	0.0286
	0.12	0.0172	0.0136	0.0099	0.0079

##### Freddie Mac

Value of Float Income per \$100 PC Face Value

$\kappa$	$\sigma$	$\lambda$			
		-0.02	-0.01	0	0.005
0.25	0.07	0.349	0.0348	0.0347	0.0346
	0.09	0.343	0.0343	0.0347	0.0346
	0.12	0.0319	0.0319	0.0320	0.0320

All other model parameters are the default (historical) values.

parameter is the market price of risk and reflects the aggregate market risk preference — that is, whether the market is more or less risk-averse. As indicated by Exhibit 7, the more risk-averse the market, the worse off are Fannie Mae servicers, reflecting the fact that the servicing contract's resale value should provide greater yield compensation for assuming the prepayment interest risk.

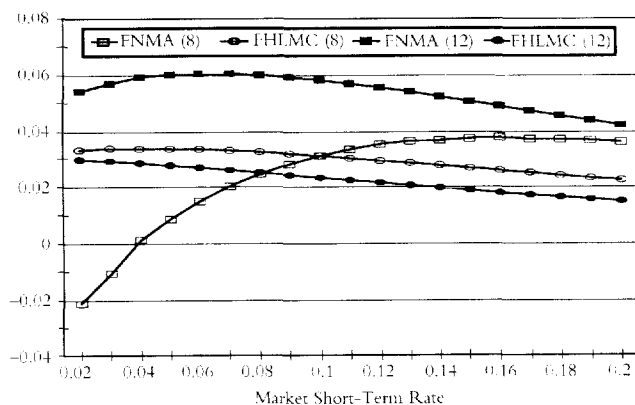
**Mortgage Pass-Through Rates.** Exhibit 6 shows the relationship of float value to mortgage pass-through rates. Mortgage refinance probability is greater on higher pass-through rate pools, because there is a greater probability that market rates will fall during the life of the pool.

Freddie Mac float benefits increase monotonically with pass-through rates, while Fannie Mae float value declines. The inverse relationship between Fannie Mae float and mortgage pass-through rates indicates that the expected prepayment interest expense outweighs the benefit of a longer float accrual period.

#### Prepayment Float Sensitivity

Exhibit 8 illustrates the relative interest rate sensitivity of the expected net prepayment float income from Fannie Mae and Freddie Mac servicing contracts. A quasi-duration measure is computed under the default interest rate environment:  $\theta = 0.08$ ,  $\kappa = 0.25$ , and  $\lambda = 0.0$ . Recall that the underlying asset is the expected net prepayment float on newly originated thirty-year fixed-rate mortgages during January 1983 through October 1995. Freddie Mac's net prepayment

EXHIBIT 6 ■ Float Value versus Market Interest Rates



float is fully insulated from prepayment interest rate risk, as reflected by the flat (zero) duration curve in Exhibit 8.

By contrast, the value of Fannie Mae's float is interest-elastic at all levels of the short-term rate. The interest rate sensitivity declines with the short-term market rate, reflecting the fact that at the higher short-term rate the risk of mortgage prepayment is reduced.

Nevertheless, servicers can attempt to lock in benefits from the longer float period under Fannie Mae contracts by taking a long (buy) position in futures contracts on an asset whose underlying value is closely related to the net float on thirty-year fixed-rate mortgages. One possibility is the ten-year Treasury note rate, since it is most highly correlated with required yields on thirty-year mortgages and reflects the likelihood of prepayments. When market rates fall, which results in increased prepayments and losses in the net float income, the losses will be offset by capital gains on the futures position.

The optimal number of futures contracts will equate the losses of float income when rates fall to the gains from off-balance sheet buying of futures when rates fall. This is given by:<sup>5</sup>

$$N_{\text{fut}} = \frac{D_{\text{float}} P_{\text{float}}}{D_{\text{fut}} P_{\text{fut}}} \quad (9)$$

where  $D_{\text{float}}$  is the float duration measure,  $P_{\text{float}}$  is the expected value of the net float income,  $D_{\text{fut}}$  is the duration of the asset to be delivered against the futures con-

tracts such as the ten-year 8% coupon T-bond, and  $P_{\text{fut}}$  is the dollar value of the initial futures contracts.

#### IV. CONCLUSION: IMPLICATIONS AND APPLICATIONS

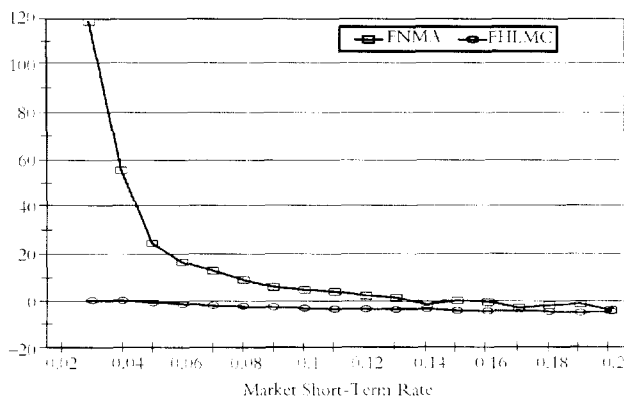
This article uses an isomorphic relationship between the cash flows of the standard prepayment float program and discount bounds to price the expected net prepayment float as an interest rate-contingent claim. We invoke the Cox-Ingersoll-Ross [1985] term-structure model to describe the underlying interest rate process and we use an empirically fitted prepayment function to derive the value of the expected net prepayment float. The valuation model provides a useful theoretical benchmark for pricing the prepayment float embedded in more general servicing contracts.

We use an explicit finite difference method to compute the historical float values for the standard Fannie Mae and Freddie Mac servicing contracts on thirty-year fixed-rate mortgages during the sample period 1983-1995. Several properties of the float with respect to interest rates environments are obtained by simulation. The simulation results suggest that Freddie Mac's float program, which insures servicers against prepayment interest obligations, is more favorable when prepayment risk is high. Specifically, the float value is greater when short-term market rates and the mean reversion level are low, and when interest rate volatility and mortgage pass-through rates are higher.

By comparison, servicers are better off with Fannie Mae's servicing contracts when short-term rates and the mean-reverting level are high, and when the market interest rate volatility is low. Under such conditions, the longer float benefit period outweighs the interest obligations from prepayments. Furthermore, servicers can lock in benefits from the longer float period by hedging the prepayment interest sensitivity with (long) positions in futures contracts. Losses in net prepayment float when interest rates fall will be offset by capital gains on the long futures positions.

Finally, the underwriter has an incentive to originate low-rate, high-point mortgages if the underwriter retains the servicing rights. Immobile borrowers are more likely to self-select into such low-rate high-point loans. Without a drop in the interest rate, these loans are less likely to be prepaid. In addition, the lower contract rate means that the market rate has to decline more for the borrower to benefit from refinancing. Meanwhile,

EXHIBIT 8 ■ Duration of Net Prepayment Float



the lower contract rate also indicates that the servicer faces a lower carrying cost in the case of prepayment. These factors increase the value of the servicing right. Because of the lower prepayment risk, they also increase the value of the mortgage and the MBS.

There are at least two other implications that are relevant from the perspective of issuers of servicing contracts. First, Fannie Mae is insulated from float risk because the risk is carried entirely by servicers. Freddie Mac, by contrast, carries the float sensitivity, but this sensitivity is off-balance sheet. The equity investors may be exposed to risk higher than they expected due to this hidden element.

Second, the float program under Fannie Mae has a potentially procyclical effect on its profitability. To illustrate, when the market interest rate rises significantly, Fannie Mae suffers a significant drop in the value of its mortgage portfolio. At the same time, significant benefits are passed to the servicers by granting them below-market rate short-term loans (through the remittance program). During a refinancing wave, Fannie Mae enjoys not only an increase in the value of its mortgage portfolio, but also substantial fees from originating and reselling MBS. In addition to reaping these profits, Fannie Mae's float program also requires its servicers to take loans at a higher-than-market rate. Therefore, Fannie Mae's prepayment float arrangement has a procyclical effect on Fannie Mae's profits — by increasing its own operating costs in bad markets and revenues in good markets — ultimately increasing volatility in profits.

## ENDNOTES

The authors wish to thank Peter Chinloy for his helpful comments.

<sup>1</sup>See Johnson [1994] for an analysis of prepayment float risk.

<sup>2</sup>McConnell [1976] describes the functions of a mortgage servicer.

<sup>3</sup>Singh and McConnell [1996] show that the value of collateralized mortgage obligation tranches is relatively insensitive to the specific term structure model employed.

<sup>4</sup>See Wolcott [1989].

<sup>5</sup>See Saunders [1997, Chapter 18].

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