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A Robust Sequential Procedure for Estimating the Number of Structural Changes in Persistence*

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Abstract

This paper proposes a new procedure for estimating the number of structural changes in the persistence of a univariate time series. While the extant literature primarily assumes (regime-wise) stationarity, our framework also allows the underlying stochastic process to switch between stationary [$I(0)$] and unit root regimes [$I(1)$]. We develop a sequential testing approach that maintains correct asymptotic size regardless of whether the regimes are $I(0)$ or $I(1)$. We also propose a novel procedure for distinguishing persistence change processes from those with pure level and/or trend shifts. Monte Carlo simulations and an application to OECD inflation rates highlight the practical usefulness of the procedures.

I. Introduction

The twin problems of testing for and estimation of structural changes in time series models have generated a vast literature in both econometrics and statistics (see Perron, 2006, for a survey). Determining the number of structural changes is a crucial component of empirical analysis from the viewpoint of model selection. An early contribution in this regard is by Yao (1988) who proposed choosing the number of breaks by minimizing the Bayesian Information Criterion (BIC). Bai and Perron (1998, BP henceforth) propose a sequential testing procedure in a general regression framework based on the sup-Wald test for structural change that involves successively applying the test to evaluate the null hypothesis of, say, l changes against the alternative of $l+1$ changes starting with $l=0$.[1] Monte Carlo evidence presented in Bai and Perron (2006) shows that the sequential testing approach dominates information criteria-based selection for a variety of data-generating processes.

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[1] A similar approach was proposed by Kejriwal and Perron (2010a) for estimating the number of changes in single-equation cointegrated models and by Kejriwal and Perron (2010b) for determining the number of trend breaks with stationary or unit root innovations.

An assumption common to the aforementioned methods of break selection is one of short memory or stationarity [referred to as $I(0)$, henceforth]. In the univariate context, this implies that the process is either $I(0)$ over the full sample (in the no break case) or $I(0)$ within regimes specified by the break dates. While convenient in theory, it rules out the possibility of a unit root [referred to as $I(1)$, henceforth] in a subsample of the data or over the whole sample. Perron (1989) showed that there is an intricate interplay between structural change and unit roots so that standard unit root tests are biased towards the unit root null if the time series is stationary around a broken deterministic trend (see Leybourne, Mills and Newbold, 2003 for a converse phenomenon). Similarly, Kejriwal and Perron (2010a) show that ignoring the possible presence of a unit root can generate spurious breaks thereby resulting in an inconsistent estimate of the number of breaks. When testing for the presence of one of these features (unit roots or structural change), it is therefore a prudent approach to allow for the possibility of the other feature.

A plethora of procedures now exists for testing structural change in persistence that allow for the possibility of a unit root under the null and/or alternative hypotheses. Harvey, Leybourne and Taylor (2006) propose test statistics for a single break based on partial sums of residuals that allow the process to be $I(1)$ or $I(0)$ throughout under the null hypothesis. Leybourne, Kim and Taylor (2007) develop recursive tests of the unit root null that can accommodate multiple changes in persistence. Kejriwal, Perron and Zhou (2013, KPZ henceforth) propose sup-Wald tests for detecting multiple persistence breaks allowing the process to be either $I(1)$ or $I(0)$ under the null hypothesis assuming the number of breaks to be known *a priori*. It thus appears relevant from a practical perspective to develop a procedure for break selection that allows the process to be either $I(1)$ or $I(0)$ in the stable case and also remains valid regardless of whether the breaks are $I(0)$ preserving or involve switches between $I(0)$ and $I(1)$ regimes.

This paper proposes a new sequential procedure for estimating the number of breaks in the persistence of a univariate time series that is robust to the presence of a unit root over the full sample or in any (asymptotically non-negligible) subsample of the data. The procedure is based on simultaneous application of two Wald-type tests for structural change, one of which assumes the process is $I(0)$ stable under the null hypothesis while the other assumes the stable $I(1)$ model as the null hypothesis. Using the intersection of the two critical regions as the relevant critical region enables the procedure to maintain correct asymptotic size regardless of whether the regimes are $I(0)$ or $I(1)$ as well as possess respectable power for detecting a variety of persistence change alternatives. The relevant asymptotic critical values are obtained from the appropriate quantiles of the single break limit distributions and tabulated for a range of trimming values. We also discuss how our procedure can be employed to address the important practical issue of distinguishing processes with pure level and/or trend breaks from those that are characterized by concurrent shifts in persistence as well. Our Monte Carlo experiments demonstrate that the advocated approach compares favourably relative to the commonly employed BP approach, especially when the data contain an $I(1)$ segment. An empirical application to OECD inflation rates further highlights the discrepancies between the two approaches, with the robust procedure selecting a smaller number of breaks when the series has one or more high persistence segments.

The rest of the paper is organized as follows. Section II lays out the persistence change model and the associated assumptions. Section III develops the proposed sequential testing procedure and its large sample properties while section IV considers a modification of the analysis to allow for deterministic trends. Section V suggests procedures for dealing with pure level and/or trend breaks. The extension to higher order serial correlation is considered in section VI. Section VII presents Monte Carlo evidence to assess the finite sample adequacy of the asymptotic approximations. Section VIII presents an application to OECD inflation rates and section IX concludes. All proofs are included in Online Appendix A with the full set of simulation results reported in Online Appendix B.

As a matter of notation, we let $\xrightarrow{p}$ denote convergence in probability, $\xrightarrow{d}$ convergence in distribution and $\Rightarrow$ weak convergence of the associated probability measures. Let $B_1(.)$ and $B_2(.)$ denote standard independent Brownian motions on $[0,1]$ and $B(.)=[B_1(.),B_2(.)]'$. Let $\tilde{B}_j^{(i)}(.)$ represent $B_j(.)$ demeaned over $[\lambda_{i-1},\lambda_i]$, that is, $\tilde{B}_j^{(i)}(r)=B_j(r)-(\lambda_i-\lambda_{i-1})^{-1}\int_{\lambda_{i-1}}^{\lambda_i}B_j$, $r\in[\lambda_{i-1},\lambda_i]$. The Brownian motions demeaned over the full sample are denoted as $\tilde{B}_j(.)=B_j(.)-\int_0^1 B_j$. All integrals of the form $\int_a^b g(r)dr$ are expressed as $\int_a^b g$.

II. The persistence change model

Consider a univariate time series y_t with data-generating process (DGP)

$$\begin{aligned} y_t &= \mu_i + u_t, \quad u_t = u_{T_{i-1}^0} + h_t \\ h_t &= \alpha_i h_{t-1} + e_t, \quad h_{T_{i-1}^0} = 0 \end{aligned} \tag{1}$$

for $t=T_{i-1}^0+1, T_{i-1}^0+2,\ldots,T_i^0; i=1,\ldots,m+1$, with $T_0^0=0$ and $T_{m+1}^0=T$, where T is the sample size. The process is therefore subject to m breaks or $m+1$ regimes with break dates $(T_1,\ldots,T_m)$. The same DGP was considered by Leybourne *et al.* (2007) and is designed to ensure that the successive $I(1)$ and $I(0)$ regimes join up at the breakpoints thereby avoiding the problem of spurious jumps to zero in u_t. We make the following assumptions on the break dates and the noise component e_t:

Assumption A1. $T_i^0=[T\lambda_i^0]$, where $0<\lambda_1^0<\cdots<\lambda_m^0<1$.

Assumption A2. $\{e_t\}$ is a martingale difference sequence with respect to $\{\mathcal{F}_t\}$, with $\mathcal{F}_t=\sigma\text{-}field\{e_s, s\leqslant t\}, E(e_t^2|\mathcal{F}_{t-1})=\sigma^2$, $\sup E(|e_t|^{4+\beta}|\mathcal{F}_{t-1})<\infty$ for some $\beta>0$.

Assumption A1 allows the development of the asymptotic theory by requiring the breakpoints to be asymptotically distinct and is standard in the structural change literature. Assumption 2 rules out serial correlation in the innovation sequence and requires conditional homoscedasticity. The case with serial correlation where e_t follows a general linear process will be considered in section VI.

From (1), we can write

$$y_t = c_i + \alpha_i y_{t-1} + e_t \tag{2}$$

where $c_i=(\mu_i+u_{T_{i-1}^0})(1-\alpha_i)$. KPZ consider tests of the null hypothesis $H_0^{(1)}$: $\alpha_i=1$ for all i. [We use the notation $H_0^{(a)}$ to denote the $I(a)$ null hypothesis, $a=0,1$]. Note that under $H_0^{(1)}, c_i=0$ for all i so that the time series follows a stable unit root process. Under the alternative hypothesis of unstable persistence, the following two models are considered depending on whether the initial regime contains a unit root or not:

Model 1a. $\alpha_i = 1$ in odd regimes and $|\alpha_i| < 1$ in even regimes; *Model 1b.* $\alpha_i = 1$ in even regimes and $|\alpha_i| < 1$ in odd regimes.

KPZ consider a variety of tests of $H_0^{(1)}$. For a given number of $m = k$ of changes, the Wald test against model 1a is defined as

$$F_{1a}(\lambda, k) = (T - k)(SSR_0^{(1)} - SSR_{1a,k}^{(1)})/[kSSR_{1a,k}^{(1)}] \quad \text{if } k \text{ is even}$$
$$F_{1a}(\lambda, k) = (T - k - 1)(SSR_0^{(1)} - SSR_{1a,k}^{(1)})/[(k + 1)SSR_{1a,k}^{(1)}] \quad \text{if } k \text{ is odd} \qquad (3)$$

In equation (3), $SSR_0^{(1)}$ denotes the sum of squared residuals under $H_0^{(1)}$ while $SSR_{1a,k}^{(1)}$ denotes the sum of squared residuals obtained from estimating (2) under the restrictions imposed by Model 1a. The test F_{1b} is defined similarly. We denote the corresponding alternative hypotheses as $H_{1a,k}^{(1)}$ and $H_{1b,k}^{(1)}$ respectively. For some small positive ϵ, define $\Lambda_\epsilon^k = \{\lambda : |\lambda_{i+1} - \lambda_i| \geqslant \epsilon, \lambda_1 \geqslant \epsilon, \lambda_k \leqslant 1 - \epsilon, i = 1, \ldots, k - 1\}$. The sup-Wald tests are then defined as $F_{1a}(k) = \sup_{\lambda \in \Lambda_\epsilon^k} F_{1a}(\lambda, k)$ and $F_{1b}(k) = \sup_{\lambda \in \Lambda_\epsilon^k} F_{1b}(\lambda, k)$. When the persistence of the initial regime is assumed to be unknown, the relevant test is $W_1(k) = \max[F_{1a}(k), F_{1b}(k)]$. Finally, when the number of breaks is unknown up to some maximal value A, the statistic is $Wmax_1 = \max_{1 \leqslant k \leqslant A} W_1(k)$.

The stable $I(0)$ null can be tested by employing the BP procedure. Specifically, consider testing $H_0^{(0)}$: $c_i = c, \alpha_i = \alpha$, for all i with $|\alpha| < 1$ in equation (2) with $c_i = \mu_i(1 - \alpha_i)$. The relevant alternative hypothesis within the BP framework is $H_{1,k}^{(0)}$: $\alpha_1 \neq \alpha_2 \neq \ldots \neq \alpha_{k+1}$, $|\alpha_i| < 1, i = 1, \ldots, k + 1$. The time series is thus regime-wise $I(0)$ under $H_{1,k}^{(0)}$. The BP test for $m = k$ changes is

$$G_1(\lambda, k) = [T - 2(k + 1)](SSR_0^{(0)} - SSR_{1,k}^{(0)})/[kSSR_{1,k}^{(0)}] \qquad (4)$$

In equation (4), $SSR_0^{(0)}$ denotes the sum of squared residuals under $H_0^{(0)}$ while $SSR_{1,k}^{(0)}$ denotes the sum of squared residuals obtained from *unrestricted* OLS estimation of (2). The BP test is then defined as $G_1(k) = \sup_{\lambda \in \Lambda_\epsilon^k} G_1(\lambda, k)$. When the number of breaks is unknown, the relevant test statistic is $UDmax_1 = \max_{1 \leqslant k \leqslant A} G_1(k)$. For details on the computation of the KPZ and BP test statistics, we refer the reader to the original articles.

III. A robust sequential procedure

The KPZ tests are based on the stable $I(1)$ null hypothesis and diverge to positive infinity when the process is stable $I(0)$ while the BP test does not have correct asymptotic size when the process is stable $I(1)$. Let $H_0 = H_0^{(1)} \cup H_0^{(0)}$. Define the following test statistics:

$$H_1(k, \eta) = \min\left[W_1(k), \frac{cv_{w,k}(\eta)}{cv_{g,k}(\eta)} G_1(k)\right], \quad Hmax_1(\eta) = \min\left[Wmax_1, \frac{cv_{w,\max}(\eta)}{cv_{g,\max}(\eta)} UDmax_1\right]$$

where, at level $\eta, cv_{w,k}(\eta), cv_{g,k}(\eta), cv_{w,\max}(\eta), and cv_{g,\max}(\eta)$ are the critical values of the tests $W_1(k), G_1(k), Wmax_1$ and $UDmax_1$ respectively. In order to control asymptotic size under H_0, KPZ recommend employing the following decision rule for a fixed number of breaks k:

$$\text{`Reject } H_0 \quad \text{if } H_1(k,\eta) > cv_{w,k}(\eta)\text{'} \tag{5}$$

(5) is equivalent to rejecting H_0 when both $W_1(k)$ and $G_1(k)$ reject. When the number of breaks is unknown, the relevant decision rule is

$$\text{`Reject } H_0 \quad \text{if } Hmax_1(\eta) > cv_{w,\max}(\eta)\text{'} \tag{6}$$

Consider using decision rule (5) with a given nominal level η for each of $W_1(k)$ and $G_1(k)$. Since $W_1(k)$ and $G_1(k)$ are correctly sized under $H_0^{(1)}$ and $H_0^{(0)}$ respectively, it follows that $H_1(k,\eta)$ has asymptotic size at most η under H_0. A similar argument holds for (6).

We now develop a sequential test of the null hypothesis of l breaks against the alternative hypothesis of $(l+1)$ breaks in equation (2). First, we obtain the estimates of the break dates $(\hat{T}_1,\ldots,\hat{T}_l)$ as global minimizers of the sum of squared residuals from the *unrestricted* model with l breaks (i.e. $SSR_{1,l}^{(0)}$) estimated by least squares.

Next, we test for the presence of an additional break in each of the $(l+1)$ segments obtained using the estimated partition $(\hat{T}_1,\ldots,\hat{T}_l)$. Let $\eta_{l+1}=1-(1-\eta)^{1/(l+1)}$. Define

$$H_1^{(i)}(1,\eta_{l+1})=\min\left[W_1^{(i)}(1),\frac{cv_{w,1}(\eta_{l+1})}{cv_{g,1}(\eta_{l+1})}G_1^{(i)}(1)\right],\quad H_1(l+1\mid l)=\max_{1\leqslant i\leqslant l+1}\{H_1^{(i)}(1,\eta_{l+1})\}$$

where $W_1^{(i)}(1)$ denotes the $W_1(1)$ test computed using data in the (estimated) regime i, that is, $[\hat{T}_{i-1}+1,\hat{T}_i]$ and $G_1^{(i)}(1)$ denotes the $G_1(1)$ test computed using the observations in $[\hat{T}_{i-1}+1,\hat{T}_i]$. We conclude in favour of a model with $(l+1)$ breaks if

$$H_1(l+1\mid l) > cv_{w,1}(\eta_{l+1}) \tag{7}$$

The test thus amounts to the computation of $(l+1)$ tests of the null hypothesis of no change vs. the alternative hypothesis of a single change and assessing whether their maximum is sufficiently large. The threshold value $cv_{w,1}(\eta_{l+1})$ is the $(1-\eta_{l+1})$-th quantile of the limit distribution of $W_1(1)$. The following result shows that the decision rule (7) has asymptotic size at most η under the null hypothesis of l breaks.

Theorem 1. Let Assumptions A1 and A2 hold. Under the null hypothesis that the true number of breaks is l, we have $\lim_{T\to\infty} P(H_1(l+1|l) > cv_{w,1}(\eta_{l+1}))\leqslant\eta$.

This result uses the fact that the estimated break fractions $(\hat{T}_1/T,\ldots,\hat{T}_l/T)$ converge at rate T regardless of the direction of change in persistence. While BP establish the T consistency of the estimated break fractions in regime-wise stationary models, Chong (2001); Kejriwal and Perron (2012) show, respectively, a similar result for a switch from an $I(0)$ to an $I(1)$ regime and from an $I(1)$ to an $I(0)$ regime.

The test based on $H_1(l+1|l)$ can be used to provide an estimate of the number of breaks $\hat{m}$ in the following way:

(i) First, apply the decision rule (6) that tests the null hypothesis H_0 against an unknown number of breaks. If a non-rejection is obtained, set $\hat{m}=0$ and the procedure stops. Otherwise, go to step 2.
(ii) Upon a rejection in step 1, use the rule (7) with $l=1$ to determine if there is more than one break. This process is repeated by increasing l sequentially until the test fails to reject the null hypothesis of no additional structural breaks.
(iii) The estimate $\hat{m}$ is then obtained as the number of rejections.

The following result shows that the proposed algorithm guarantees that the probability of selecting the true number of breaks is at least $(1-\eta)$ in large samples:

Theorem 2. Let Assumptions A1 and A2 hold. Let m be the true number of breaks and $\hat{m}$ be the estimated number of breaks obtained using nominal level η. Then $\lim_{T\to\infty}P(\hat{m}=m)\geqslant 1-\eta$.

The sequential procedure can be made consistent by allowing the significance level of the tests $Hmax_1$ and $H_1(l+1|l)$ to decrease to zero at a suitable rate as the sample size increases. It can be shown that if the true number of breaks is at least $l+1$, $Hmax_1$ and $H_1(l+1|l)$ diverge at rate $O_p(T)$. Thus, if the critical values $cv_{w,1}, cv_{g,1}, cv_{w,\max}, cv_{g,\max}$ are allowed to increase at rate $O_p(T^{1-\varepsilon})$, $0<\varepsilon<1$, the size of the tests converges to zero at T increases while ensuring their consistency under the alternative. We thus have the following corollary whose proof is similar to that of Hosoya (1989) and is thus omitted:

Corollary 1. Let m be the true number of breaks and $\hat{m}$ be the number of breaks obtained using the sequential procedure based on the test statistic $H_1(l+1|l)$ applied with significance level η_T. Consider a sequence of critical values $cv_{w,1}=c_1T^{1-\varepsilon}, cv_{g,1}=c_2T^{1-\varepsilon}, cv_{w,\max}=c_{\max}T^{1-\varepsilon}$ ($c_1, c_2, c_{\max}$ are positive constants) so that η_T converges to zero while ensuring that $H_1(l+1|l)$ remains consistent. Then, $P(\hat{m}=m)\to 1$ as $T\to\infty$.

Remark 1. Using the rule (6) in the first step instead of the $H_1(1|0)$ statistic is motivated by the fact that single break tests can suffer from low power in the presence of multiple breaks. Power can potentially still be an issue with more than three breaks, for example, the DGP has four breaks with two breaks on either side of the first estimated breakpoint.

Remark 2. Theorem 2 and Corollary 1 hold not only under the alternative hypotheses $H^{(1)}_{1a,m}$, $H^{(1)}_{1b,m}$ and $H^{(0)}_{1,m}$ but also under more general alternatives where the process involves a mix of $I(1)$ and $I(0)$ regimes with possibly adjacent $I(0)$ regimes.

Remark 3. While Theorem 2 suggests that the probability of break selection can be guaranteed to exceed any preassigned value by choosing a sufficiently small η, it must be borne in mind that it is a large sample result that uses the fact that the test $H_1(l+1|l)$ is consistent under the alternative hypothesis of at least $(l+1)$ breaks. In finite samples, however, the power of the test depends on the significance level used and using too small a level can lead to underestimating the true break number.

IV. Deterministic trends

This section discusses how the sequential procedure proposed in section III can be extended to allow for the presence of deterministic trends. We consider an extension of (1) that includes the possibility of m breaks in the deterministic trend:

$$y_t=\mu_0+\beta_0 t+\sum_{j=1}^{m}\mu_j DU_{jt}+\sum_{j=1}^{m}\beta_j DT_{jt}+u_t \tag{8}$$

where $DU_{jt} = I(t > T_j^0), DT_{jt} = I(t > T_j^0)(t - T_j^0); j = 1, \ldots, m$, and u_t as specified in equation (1). The DGP (8) can be expressed as

$$y_t = c_i + b_i t + \alpha_i y_{t-1} + e_t \tag{9}$$

The difference between (2) and (9) is the presence of the deterministic trend in the latter and that the intercept c_i is now a function of the trend parameters appearing in equation (8). KPZ propose tests of the null hypothesis $\tilde{H}_0^{(1)}$: $c_i = c, \alpha_i = 1$ for all i in equation (9). Under $\tilde{H}_0^{(1)}, b_i = 0$ for all i so that the process follows a stable unit root process with possible drift. As in the trendless case, KPZ consider two models (denoted as 2a and 2b) under the alternative hypothesis depending on whether the initial regime is trend or difference stationary. The test statistics are accordingly denoted by $F_{2a}(\lambda, k), F_{2b}(\lambda, k), W_2(k)$ and $Wmax_2$.

We now turn to testing the null of a stable trend stationary process, that is, $\tilde{H}_0^{(0)}$: $c_i = c, b_i = b, \alpha_i = \alpha$ for all i where $|\alpha| < 1$ in equation (9). The test statistic for a fixed number $m = k$ changes is based on

$$G_2(\lambda, k) = [T - 3(k+1)](\widetilde{SSR}_0^{(0)} - SSR_{2,k}^{(0)})/[kSSR_{2,k}^{(0)}] \tag{10}$$

In equation (10), $\widetilde{SSR}_0^{(0)}$ denotes the sum of squared residuals under $\tilde{H}_0^{(0)}$, that is, that obtained from OLS estimation of (9) subject to the restrictions $c_i = c, b_i = b, \alpha_i = \alpha$ for all i. The quantity $SSR_{2,k}^{(0)}$ denotes the sum of squared residuals obtained from *unrestricted* OLS estimation of (9). The test statistic is then defined as $G_2(k) = \sup_{\lambda \in \Lambda_\epsilon^k} G_2(\lambda, k)$. When the number of breaks is unknown, the relevant test statistic is $UDmax_2 = \max_{1 \leqslant k \leqslant A} G_2(k)$.

The limit distributions of $G_2(k)$ and $UDmax_2$ are *not* the same as those of $G_1(k)$ and $UDmax_1$ and therefore the BP critical values cannot be used. The relevant limit result is stated in the following theorem:

Theorem 3. Suppose Assumptions A1 and A2 hold. Let $F(r) = (1, r)'$, $r \in [0, 1]$. Under $\tilde{H}_0^{(0)}$,

$$G_2(k) \Rightarrow G_2{}^*(k), \quad UDmax_2 \Rightarrow \max_{1 \leqslant k \leqslant A} G_2{}^*(k)$$

The expression for the limiting random variable $G_2^*(.)$ is provided in Appendix B. It depends on $F(.)$ and $B_1(.)$ but not on any nuisance parameters (given ϵ). While the limit is different from that in BP, asymptotic critical values can be obtained through simulation.

Remark 4. The sequential procedure described in section III can be applied in the trending case with $G_1(1), W_1(1), UDmax_1$ and $Wmax_1$ replaced by $G_2(1), W_2(1), UDmax_2$ and $Wmax_2$, respectively. Theorems 1 and 2 still hold for this modified sequential procedure.

Remark 5. Bai (1999) proposes an alternative likelihood-ratio–based procedure allowing for trending regressors based on estimating l breaks simultaneously under the null and estimating $(l+1)$ breaks simultaneously under the alternative. While his critical values depend on the true breakpoints and thus must be computed on a case-by-case basis, ours can be tabulated for general use. His procedure does not allow for the possibility of a unit root.

V. Structural breaks in level and trend

This section addresses the empirically important issue of distinguishing between processes with pure level shifts and/or trend breaks from those where these breaks are accompanied by concurrent shifts in persistence. Appendix B provides Monte Carlo evidence on the finite sample performance of these procedures.

Breaks in level

The KPZ and BP tests are consistent against processes with pure shifts in level but a stable $I(0)$ persistence parameter and so therefore cannot be directly used to distinguish between processes characterized by level shifts only and those that are characterized by concurrent shifts in level and persistence. Our proposed procedure for distinguishing between these two possibilities is related to the two-step approach taken in Hsu and Kuan (2001) for disentangling a slope change from a level shift in a time trend model with stable $I(0)$ errors. In a first step, the joint null of stability in both coefficients is tested and, conditional on a rejection, the break date is estimated by minimizing the sum of squared residuals and used in a second step to test the stability of the intercept or trend coefficient, while allowing the other coefficient to change.

Our approach is based on the fact that the estimated number of breaks obtained from applying the robust sequential procedure is still consistent even if the process is only subject to shifts in level. Furthermore, the estimated breakpoints $\hat{\lambda} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_{\hat{m}})'$ obtained by minimizing the global sum of squared residuals from the *unrestricted* model (2) are T consistent for the corresponding true breakpoints regardless of whether the level shifts are accompanied by concurrent shifts in persistence (Bai, 1994; BP). Specifically, consider the standard Wald statistic for testing $\alpha_i = \alpha$ for all i in the model

$$y_t = c_i + \alpha_i y_{t-1} + e_t, \quad t \in [\hat{T}_{i-1} + 1, \hat{T}_i]; i = 1, \ldots, m+1 \tag{11}$$

Denote the resulting statistic at $W^*(m)$. The following result establishes the limit distribution of $W^*(m)$ under the null hypothesis that the process has a stable $I(0)$ persistence parameter with m shifts in level:

Theorem 4. Suppose Assumptions A1 and A2 hold. Under the null hypothesis $\alpha_i = \alpha \forall i$ where $|\alpha| < 1$, $W^*(m) \xrightarrow{d} \chi^2(m)$.

We thus recommend the following three-step procedure. First, we determine the number of breaks ($\hat{m}$) applying the robust sequential procedure proposed in section III. Second, conditional on $\hat{m}$ and the corresponding estimates of the breakpoints $(\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_{\hat{m}})$ obtained from the unrestricted model (2), we compute the Wald statistic for testing the stable $I(0)$ null hypothesis (i.e. constancy of α_i over all i in equation (11) while allowing the intercept to vary across the $(\hat{m} + 1)$ regimes. Third, the null hypothesis of stable $I(0)$ persistence is rejected if the Wald statistic is significant at the specified level where the critical value is obtained from the $\chi^2(\hat{m})$ distribution. Otherwise, the null is not rejected and we conclude in favour of a model with pure level shifts.

Breaks in trend

The trending case is further complicated by the fact that with pure trend breaks, the process can be either trend stationary or difference stationary, that is, the persistence parameter can be either $I(1)$ stable or $I(0)$ stable. We exploit the fact that Theorem 2 remains valid for the robust sequential procedure even when the process is subject to trend breaks only, given the joint nature of the null hypotheses. Furthermore, the estimates of the break-points obtained by global minimization of the sum of squared residuals in the *unrestricted* model (9) that allows concurrent trend and persistence breaks are T consistent regardless of whether the trend breaks are accompanied by shifts in persistence in the true DGP. In the pure trend break case, this result follows from the results in Perron and Zhu (2005) while in the concurrent case, it follows from the results in Chong (2001); Kejriwal and Perron (2012). Based on the estimated break number and the corresponding break-points, the null hypothesis of constant persistence can be evaluated by testing for structural change in the persistence parameter while allowing the trend parameters to change across regimes.

For testing $I(0)$ stability, we consider the standard Wald statistic for testing $\alpha_i = \alpha$ for all i in the model

$$y_t = c_i + b_i t + \alpha_i y_{t-1} + e_t, \quad t \in [\hat{T}_{i-1} + 1, \hat{T}_i], i = 1, \ldots, m+1 \tag{12}$$

Denote the resulting statistic as $\tilde{W}_0(m)$. For testing $I(1)$ stability, we compute the Wald statistic based on the difference between the restricted and unrestricted sum of squared residuals where the former is obtained by estimating the restricted model

$$\Delta y_t = c_i + e_t, \quad t \in [\hat{T}_{i-1} + 1, \hat{T}_i], i = 1, \ldots, m+1 \tag{13}$$

while the latter is obtained by estimating the unrestricted model (12). We denote this statistic as $\tilde{W}_1(m)$. The following theorem states the limit distributions of $\tilde{W}_0(m)$ and $\tilde{W}_1(m)$ under the respective null hypotheses:

Theorem 5. Suppose that Assumptions A1 and A2 hold. Let $F(r) = (1, r)'$. Then, under the null hypothesis of m trend breaks,

(i) If $\alpha_i = \alpha \quad \forall i$ with $|\alpha| < 1$, $\tilde{W}_0(m) \xrightarrow{d} \chi^2(m)$.
(ii) If $\alpha_i = 1 \quad \forall i$, $\tilde{W}_1(m) \Rightarrow \tilde{W}_1^*(m, \lambda^0)$.

While the statistic $\tilde{W}_0(m)$ has a standard chi-squared limiting distribution, $\tilde{W}_1(m)$ has a non-standard limit distribution depending on m and $\lambda^0 = (\lambda_1^0, \ldots, \lambda_m^0)'$. The expression for the limiting random variable $\tilde{W}_1^*(.,.)$ is provided in Appendix B. To obtain the critical values, the limit can be approximated via Monte Carlo simulation by replacing λ^0 with the estimated break fractions $\hat{\lambda} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_{\hat{m}})'$ obtained from the unrestricted model (12) and replacing the Wiener process $B_1(.)$ by partial sums of i.i.d. standard normal deviates. A three-step sequential approach akin to that described in the previous subsection can then be implemented with the modification that in the final step the null hypothesis of constant persistence [$I(1)$ or $I(0)$] is rejected if *both* statistics $\tilde{W}_0(\hat{m})$ and $\tilde{W}_1(\hat{m})$ are significant at the specified level.

VI. Higher order serial correlation

We now provide an extension of model (1) that allows the time series $\{y_t\}$ to be generated by an AR(p) process. Specifically, we make the following assumption that allows at most one unit root in each regime while requiring all remaining roots to be stationary.

Assumption A3 The errors h_t in equation (1) are generated as

$$\left.\begin{aligned} h_t &= \alpha_i h_{t-1} + \sum_{j=1}^{p-1} \pi_{ij} \Delta h_{t-j} + e_t \\ h_{T_{i-1}^0} &= \ldots = h_{T_{i-1}^0 - p+1} = 0 \end{aligned}\right\} \begin{aligned} t &= T_{i-1}^0 + 1, T_{i-1}^0 + 2, \ldots, T_i^0; \\ i &= 1, \ldots, m+1 \end{aligned} \tag{14}$$

where all roots of the polynomial $\pi(L) = 1 - \pi_{i1}L - \pi_{i2}L^2 - \ldots - \pi_{i,p-1}L^{p-1}$ lie outside the unit circle and $\{e_t\}$ satisfies Assumption A2.

In the non-trending case, $y_t = \mu_i + u_t$ which leads to the test regression

$$\Delta y_t = c_i + (\alpha_i - 1)y_{t-1} + \sum_{j=1}^{p-1} \pi_j \Delta y_{t-j} + e_t{}^* \tag{15}$$

where $c_i = (\mu_i + u_{T_{i-1}^0})(1 - \alpha_i)$ and e_t^* is the regression error. In the trending case, we augment (15) with a deterministic time trend regressor. The test statistics in the serially correlated case are constructed in a similar way as in section III except that the relevant test regression is now (15) instead of (2). Under Assumptions A1–A3, Theorems 1–5 are expected to hold for the KPZ and BP test statistics computed from (15). This follows from the fact that all limit results for the test statistics (under the null and alternative hypotheses considered in the AR(1) case) remain valid in the general case as long as the statistics are computed from (15) [see Theorems 2 and 3 in KPZ].

The true lag order p is assumed unknown so an important practical issue regarding the implementation of the sequential approach concerns the choice of the lag order (say l_T) in estimating the specification (15). Based on extensive simulation experiments, we found the following approach to be both computationally efficient as well as deliver robust results with respect to selecting the true number of breaks. First, we determine the lag length using BIC based on the stable $I(0)$ and stable $I(1)$ null hypotheses respectively. The maximum of the two lag lengths is then used to compute the $Wmax_1$ and $UDmax_1$ (or $Wmax_2$ and $UDmax_2$ in the trending case) tests in step 1 of the sequential procedure. Second, upon a rejection, the *unrestricted* single break model is estimated over the full sample for each allowable lag length (zero to, say, $l_{\max}$), whereby all regression coefficients including those of the lagged differences are allowed to change across regimes. Choose the lag length as the minimizer of the BIC over $[0, l_{\max}]$. Third, the lag length thus determined is used to compute the test statistics in the two regimes specified by the estimated break date. Subsequently, at each stage 'j' of the procedure, choose the lag length by minimizing the BIC across permissible lag lengths where each model is estimated by global minimization of the sum of squared residuals allowing for j breaks. The test statistics in the $(j+1)$ estimated regimes are then computed using this choice. In this way, the choice of the lag length adapts to the null hypothesis under consideration at each step (j vs. $(j+1)$ breaks). This approach to lag selection was observed to dominate an approach based on a fixed number of lags as well as one where the lag choice was made once and for all under either the stable $I(0)$ or $I(1)$ null.

VII. Monte Carlo evidence: Summary

This section summarizes results from a set of Monte Carlo experiments designed to evaluate the finite sample performance of the proposed approach (denoted Seq_R) relative to the BP approach (denoted Seq_{BP}). The full set of results is available in Appendix B. Asymptotic critical values for implementing the sequential test of l vs. $(l+1)$ breaks $(l=0,1,\ldots,5)$ are obtained by Monte Carlo simulation and provided in Tables 1 and 2 for three trimming choices: $\epsilon=0.15,0.20,0.25$.

We considered an AR(1) DGP for $\{y_t\}$ with zero, one and two persistence breaks respectively. The breaks either involve a switch between $I(1)$ and $I(0)$ regimes or between different $I(0)$ regimes. Three types of error structures based on N(0,1) innovations were entertained: i.i.d; AR(1); MA(1). Three sample sizes are considered: $T=200;400;600$. The sequential test (for both procedures) at each step is evaluated at the 10% nominal level (i.e. $\eta=.10$ in section III). We also considered using $\eta=.05$ but found that for $T\leqslant 400$, the underestimation probabilities were considerable and that $\eta=.10$ appeared to provide the best compromise in terms of the size-power trade-off while the two levels delivered comparable results for $T=600$. The trimming parameter was set at $\epsilon=.15$ and the maximum number of breaks at $A=5$. All experiments are based on 1000 replications. To account for serial correlation, we implement the lag selection procedure outlined in section VI with $l_{\max}=4$.[2] The principal findings are summarized as follows.

(i) In the no breaks scenario with i.i.d. errors, when the process is relatively less persistent ($\alpha\leqslant .7$), the performance of the two procedures is very similar. However, as α increases further, the differences become quite prominent with Seq_{BP} selecting at least one break far more frequently.

(ii) With at least one persistence break and i.i.d. errors, Seq_{BP} continues to be oversized as long as one regime is highly persistent. This is true even when the breaks are $I(0)$ preserving where the BP approach is asymptotically valid. On the other hand, Seq_R exhibits a more reliable performance in identifying the true model across different break locations for a given DGP.

(iii) With serially correlated errors, the relative performance of the two approaches hinges on the nature of serial correlation. While Seq_R continues to be accurate for positively correlated errors, it can be subject to substantial underestimation when a negative MA component is present. This is due to the fact that the autoregressive approximation (15) does not adequately account for the serial correlation in this case. Therefore, just as not accounting for positive serial correlation (i.e. a unit root) causes a bias in favour of a model with structural change, negative serial correlation induces a bias against a model with structural change. This issue is,

[2] Given that the sequential tests are computed from successively smaller subsamples, it is important to use a relatively small number of lags from a parsimony standpoint when constructing the various statistics. Nevertheless, we also implemented the procedure with $l_{\max}=6$ and found the results to be inferior in terms of true break selection probabilities across procedures compared to using $l_{\max}=4$.

Table 1

Asymptotic critical values of the $H_j(l+1|l)$ tests for $j=1,2$ [i.e. $cv_{w,j}(\eta_{l+1})$, $cv_{w,\max}(\eta)$]

ϵ	$1-\eta$	l = 0	1	2	3	4	5	$Wmax_1$
Non-trending case								
.15	.90	8.09	8.94	9.53	9.96	10.29	10.51	9.86
	.95	8.99	10.00	10.52	10.91	11.20	11.44	10.90
	.975	10.00	10.95	11.46	11.68	12.06	12.34	11.95
	.99	11.21	12.06	12.66	1.90	13.24	13.53	13.02
.20	.90	7.85	8.80	9.32	9.84	10.18	10.34	9.30
	.95	8.85	9.86	10.34	10.64	11.00	11.28	10.23
	.975	9.88	10.67	11.29	11.72	11.99	12.10	11.16
	.99	11.03	11.99	12.18	12.44	12.97	13.72	12.12
.25	.90	7.61	8.52	8.98	9.40	9.76	10.07	8.63
	.95	8.55	9.43	10.11	10.41	10.75	10.97	9.49
	.975	9.45	10.43	11.00	11.33	11.71	11.81	10.36
	.99	10.77	11,71	11.99	12.17	14.44	12.62	11.57

ϵ	$1-\eta$	l = 0	1	2	3	4	5	$Wmax_2$
Trending case								
.15	.90	7.28	7.96	8.39	8.72	8.91	9.10	7.71
	.95	7.98	8.74	9.11	9.56	9.72	9.80	8.43
	.975	8.75	8.75	9.84	10.26	10.60	10.66	9.18
	.99	9.73	9.73	10.60	10.75	11.12	11.18	10.07
.20	.90	7.15	7.82	8.19	8.47	8.60	8.89	7.42
	.95	7.82	8.48	8.91	9.09	9.20	9.38	8.08
	.975	8.51	9.10	9.39	9.60	9.94	10.10	8.65
	.99	9.21	9.94	10.43	10.70	10.89	11.23	9.42
.25	.90	6.97	7.66	8.07	8.37	8.55	8.67	7.14
	.95	7.69	8.38	8.68	9.02	9.16	9.27	7.82
	.975	8.41	9.03	9.30	9.44	9.64	9.94	8.51
	.99	9.16	9.64	10.16	10.69	10.85	11.23	9.22

however, ameliorated as the sample size increases and Seq_R is seen to dominate Seq_{BP} when $T \geqslant 400$. Overall, the performance of Seq_R is accurate for $T \geqslant 400$ unless a large negative MA component is present.

(iv) An empirically important issue concerns the choice of the maximum allowable number of breaks in relation to the sample size. Given that the sequential tests are implemented on subsamples of the data, allowing for too many breaks with a relatively small sample size is likely to result in size distortions/low power resulting in overestimating/underestimating the number of breaks. Based on our experiments, it appears reasonable to allow for a maximum of five breaks when $T=400$.

TABLE 2

Asymptotic critical values of the BP tests [i.e. $cv_{g,1}(\eta_{l+1})$, $cv_{g,\max}(\eta)$], trending case

		l						
ϵ	$1-\eta$	0	1	2	3	4	5	$UDmax_2$
.15	.90	10.77	12.60	13.56	14.28	14.71	15.00	11.04
	.95	14.33	15.02	15.79	16.51	16.82	16.82	12.85
	.975	14.36	15.82	16.89	17.27	17.63	18.09	14.40
	.99	16.57	17.63	18.52	18.85	19.27	19.42	16.57
.20	.90	10.43	12.20	13.27	13.95	14.54	14.88	10.59
	.95	12.23	14.01	14.96	15.63	16.26	16.59	12.40
	.975	14.12	15.68	16.60	17.13	17.47	17.87	14.13
	.99	16.34	17.47	18.27	18.85	19.42	20.03	16.34
.25	.90	10.09	11.87	12.88	13.56	14.26	14.58	10.21
	.95	11.95	13.61	14.64	15.26	15.96	16.39	12.01
	.975	13.69	15.38	16.47	16.92	17.37	17.47	13.70
	.99	15.99	17.37	18.09	18.50	19.42	19.60	16.00

VIII. Empirical application: OECD inflation persistence

This section applies the proposed methodology to analyze the nature of the inflation process for a set of OECD countries. Given the adoption of inflation targeting as the primary objective of long-run monetary policy in many countries, an issue that has been subject to intense debate among academics and policymakers is whether inflation persistence is an inherent characteristic of the economy that should be explicitly incorporated while formulating macroeconomic models. According to one view, inflation persistence is stable and high, that is, consistent with the presence of a unit root. For instance, Stock (2001); Pivetta and Reis (2007) present evidence supporting this view for the USA while O'Reilly and Whelan (2005) provide corroborating evidence for the Euro area. In contrast, another strand of the literature argues that inflation persistence is time varying and depends on the transparency and credibility of the underlying monetary policy regime. Taylor (2000); Cogley and Sargent (2001) find that both the level and persistence of US inflation has decreased over time (see Benati, 2008 for similar evidence on Euro area inflation). Yet another strand of the literature argues that inflation persistence is low and stable once one allows for time variation in the level of inflation (Levin and Piger, 2004).

The bulk of the empirical literature addressing this debate has focused on the application of unit root tests with or without allowing for mean shifts and standard structural break tests (Andrews, 1993; Bai and Perron, 1998) on the mean and/or persistence parameters. Two recent contributions in this context are Noriega, Capistrán and Ramos-Francia (2013); Belaire-Franch (2019). The former is based on the application of the methodology advocated by Leybourne *et al.* (2007) which restricts the null to be stable $I(1)$ while the latter employs the original KPZ procedure which does not address the issue of model selection, that is, the number and nature of breaks driving the time series.

Our empirical analysis is based on monthly CPI inflation data for 19 OECD countries used in Noriega *et al.* (2013); Belaire-Franch (2019). The data span the period

1960:01–2008:06 so that $T = 582$, except for Germany and Korea where the starting point is 1960:02.[3] The inflation rate is computed as $i_t = 1200(\ln P_t - \ln P_{t-1})$, where P_t denotes the CPI at time t. All inflation rates are seasonally unadjusted.[4] The results are reported in Table 3. The estimates of the number of breaks from the robust and BP approaches are denoted $\hat{m}_R$ and $\hat{m}_{BP}$ respectively. We also report the largest least squares persistence estimate across the $(\hat{m}_R + 1)$ regimes demarcated by the estimated break dates, the corresponding median unbiased estimate and the 90% confidence band computed using Hansen's (1999) grid bootstrap procedure based on 999 bootstrap replications and 200 grid points.[5] The parameter estimates were obtained from a specification in which the lag length was chosen using the BIC with a maximum permissible length of 12. Conditional on the estimate $\hat{m}_R$, we conducted tests for the null hypothesis of pure mean shifts as discussed in section V in order to determine the nature of the breaks. Finally, the last two columns report the model selected by $\hat{m}_R$ and the estimated break dates respectively.

Several features of the findings are noteworthy. First, Seq_{BP} selects more breaks ($\hat{m}_{BP} > \hat{m}_R$) in 13 out of the 19 countries. Of these, the BP approach estimated one additional break for Italy and Germany while at least two additional breaks for the other 11 countries. Second, comparing the cases where $\hat{m}_{BP} = \hat{m}_R$ with those where $\hat{m}_{BP} > \hat{m}_R$, a clear pattern emerges. For the six countries where the two procedures concur, the largest least squares persistence estimate lies in the range [.00,.69] while the median unbiased estimate lies in [.00,.73]. In sharp contrast, in 10 out of the 13 countries where the procedures disagree, the corresponding range for the former is [.82,.91] while that for the latter is [.87,.98]. Furthermore, the 90% confidence bands include unity in nine of the ten countries (the exception being USA). The empirical evidence is therefore consistent with the evidence presented earlier which indicated that the BP approach is likely to overestimate the number of breaks in the presence of a highly persistent segment in the time series. Third, the data provide evidence in favour of a pure mean shifts model in only six countries.[6] The implication is that one might obtain misleading evidence on the degree of inflation persistence when relying on a model that only allows for breaks in level as opposed to a model that allows for a break in both level and persistence.

Turning to model selection, we classify a regime as $I(1)$ if the 90% confidence band for the persistence parameter includes unity and $I(0)$ otherwise.[7] The results show that the

[3] The data set used in Noriega *et al.* (2013) and Belaire-Franch (2019) contains five additional OECD countries (Australia, New Zealand, Hungary, Iceland, Ireland) which we excluded from our analysis as the data for these countries were available for considerably shorter time spans.

[4] We prefer to use seasonally unadjusted rates since commonly used adjustment procedures such as Census X-11 or X-12 can have adverse effects on the power of structural change tests by smoothing the time series of interest (see Ghysels and Perron, 1996).

[5] To save space, we do not report the autoregressive estimates for each regime. The results are available upon request.

[6] For Canada, the null hypothesis of a pure mean shift model was not rejected at the 10% level. However, the 90% confidence band for the persistence parameter based on this model was found to be [.81,1.03] which includes unity. We therefore also estimated a model that allows a break in both level and persistence and obtained 90% confidence bands for the first and second regimes as [.86,1.05] and [.07,.39] respectively (with corresponding median unbiased estimates .98 and .23). Based on this finding, our preferred specification for Canada is one that allows a concurrent break in mean and persistence.

[7] For brevity, we do not present the persistence estimates for each regime. The full set of results is available upon request.

TABLE 3

Break selection in OECD inflation rates

Country (1)	$\hat{m}_R$ (2)	$\hat{m}_{BP}$ (3)	*PMS* (4)	*LARS* (5)	*90% Band* (6)	*MUB* (7)	*Selected model* (8)	*Break dates* (9)
Austria	3	5	Yes	.13	[-.11,.46]	.17	I(0) with three mean shifts	70:3; 85:3; 96:1
Belgium	1	5	No	.82	[.74,1.03]	.87	I(0) - I(1)	74:9
Canada	1	5	No	.91	[.86,1.05]	.98	I(1) - I(0)	99:2
Finland	1	3	No	.87	[.82,1.01]	.90	I(0) - I(1)	73:5
France	2	2	No	.65	[.50,1.02]	.72	I(0) - I(1) - I(0)	73:1; 83:4
Germany	4	5	No	.46	[.29,.72]	.53	I(0) - I(0) - I(0) - I(0) - I(0)	70:9; 82:2; 92:6; 00:12
Greece	1	4	No	.87	[.81,1.04]	.92	I(0) - I(1)	70:8
Italy	4	5	No	.66	[.58,.84]	.71	I(0) - I(0) - I(0) - I(0) - I(0)	72:6; 79:9; 87:7; 96:5
Japan	1	5	No	.88	[.82,1.06]	.97	I(0) - I(1)	71:11
Korea	1	1	Yes	.14	[.06,.23]	.14	I(0) with one mean shift	81:9
Luxembourg	1	5	No	.87	[.80,1.06]	.95	I(1) - I(0)	99:1
Netherlands	5	5	Yes	.00	[−.04,.05]	.00	I(0) with five mean shifts	67:12; 75:2; 82:4; 89:6; 01:1
Norway	2	2	Yes	.30	[.10,.56]	.33	I(0) with two mean shifts	69:12; 88:9
Portugal	5	5	Yes	.13	[.07,.20]	.14	I(0) with five mean shifts	70:10; 78:2; 85:4; 92:6; 99:10
Spain	0	4	No	.88	[.82,1.04]	.92	I(1)	—
Sweden	0	2	No	.82	[.74,1.03]	.87	I(1)	—
Switzerland	1	1	Yes	.69	[.57,.89]	.73	I(0) with one mean shift	93:4
UK	1	3	No	.87	[.79,1.08]	.98	I(1) - I(0)	81:4
USA	1	4	No	.87	[.81,.98]	.89	I(0) - I(0)	81:9

Notes: (1) the country, (2) the number of breaks selected by $Seq_R[\hat{m}_R]$, (3) the number of breaks selected by $Seq_{BP}[\hat{m}_{BP}]$, (4) whether the series is only subject to pure mean shifts (PMS), (5) largest sutoregressive sum (LARS) estimate across regimes, (6) 90% confidence band computed using Hansen's (1999) grid bootstrap, (7) the median unbiased estimate (MUB) based on the grid bootstrap, (8) the model selected by $Seq_R[\hat{m}_R]$, (9) break date estimates.

data are consistent with the presence of an $I(1)$ segment in ten countries. Interestingly, in all five countries that experienced a switch from $I(0)$ to $I(1)$ behavior, the estimated break date lies between the late 1960s and the mid 1970s, a period that is commonly believed to be associated with high levels of inflation as well as high inflation persistence (the so called 'Great Inflation').[8] On the other hand, $I(1)$-$I(0)$ switches (or high to low $I(0)$ changes) are estimated to occur in the 1980s and 1990s, a period where inflation targeting was adopted by Central banks in several countries as the principal objective of monetary policy. Interestingly, the evidence for USA favours an inflation process that is subject to a single $I(0)$-preserving break at 1981:09. The least squares autoregressive estimate (the median-unbiased estimate) declined from .87 (.89) in the pre-break regime to .27 (.28) in the post-break regime with corresponding 90% bands [.81,.98] and [.18,.38] respectively. This

[8] Even for Italy with $I(0)$ preserving persistence changes in the 1970s, these changes were associated with a movement from a low persistence regime to a high persistence one.

pattern is consistent with evidence suggesting the role of an aggressive policy stance taken by then Federal Reserve Chairman Paul Volcker to combat inflation (see, e.g., Cogley and Sargent, 2001). In summary, the empirical evidence suggests that the proposed procedure offers a useful alternative to existing approaches in facilitating reliable determination of the number of breaks in the level and/or persistence of economic time series.

IX. Conclusion

This paper proposes a new sequential procedure for estimating the number of breaks in the persistence of a univariate time series, when it is not known *a priori* whether the regimes are $I(1)$ or $I(0)$. Two extensions are worth noting. First, it would be useful to extend our approach to the case of error heteroscedasticity given the pervasive evidence on volatility shifts in economic time series (McConnell and Perez-Quiros, 2000). A wild bootstrap approach can potentially be employed to construct versions of the KPZ and BP tests that are robust to volatility shifts, a topic of ongoing research by the author (Kejriwal and Yu, 2019). Second, a more general framework for assessing persistence change that allows the process to be stationary long memory in some regimes (i.e. $I(d), d > 0$) should pave the way for a wide range of interesting empirical applications.

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix A. Proofs.
Appendix B. Monte Carlo Results.